Neutron star binaries:
Information obtained with only GW

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See:
  Wysocki et al arxiv:2001.01747
  Nepal et al CQG 2020
Outline

• Binary neutron stars
  • Combining many events
  • EOS constraints

• Asymmetric binaries (“BH-NS”)
  • Single exceptional events by example: Learning from 0814
  • How well can we constrain parameters of a precessing binary?
Context: Probing NS populations

- Many multi messenger probes of NS / BNS properties, like masses, spins, ...
  - Pulsar observations most numerous -> mass distribution

![Mass distribution](image)

**Ozel and Freire 1603.02698**

![NS mass distribution](image)

**Alsing et al 2018**
How our measurements correlate mass/spin/tides

- Prior knowledge about NS spins strongly impacts interpretation of NS masses (and thereby tidal constraints)

![Graph showing correlation between mass, spin, and tidal effects](image)

Abbott et al 2020 (190425z)
Synthetic GW survey of NS-NS mergers

- What happens if we combine all information, mass/spin/tides?
  - We correctly recover the mass distribution

Synthetic GW survey of NS-NS mergers

- What happens if we combine all information, mass and spin?
- & spin, and identify multiple populations

Nuclear equation of state

• (Chirp) mass (one data point = consistent with galactic)
• (Aligned part of) spin small, consistent with galactic
• Tides and the nuclear EOS

$\require{cancel}
\Lambda = \Lambda(m)$

Abbott et al 1805.11581
Nuclear equation of state

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- Tides and the nuclear EOS

\[ \Lambda = \Lambda(m) \]
Context: Probing NS matter properties

- NS size & maximum mass set by the nuclear EOS

- Other observations probe nature of NS matter (e.g., size/tides)

Raaijmakers et al 2019 [see also Riley et al 2019, Miller et al 2019, … ]
[talks earlier in this session, and Steiner et al in session J]
Synthetic GW survey of NS-NS mergers

- What happens if we combine all information, mass and spin?
- and the nuclear EOS
How important is joint mass/spin/EOS inference?

- Consider several small surveys of 1, 5, 10, ... BNS mergers
  - Using a mass/spin model that is compatible with the data, recover EOS
How important is joint mass/spin/EOS inference?

- Consider several small surveys of 1, 5, 10, … BNS mergers
  - Using a mass/spin model that is oversimplified, we introduce biases in the recovered EOS
Neutron stars: Conclusions

- Joint inference critical
  - Nature & our measurements correlate interesting questions [e.g., mass/spin/tide degeneracy]
  - Do better at every challenge using maximal information … and can misinterpret if you don’t fit all properties together
  - Going forward: helps with ambiguous events (GW190425z)

- Joint inference practical
  - Likelihood-based helps overcome computational hurdles, without reinventing GW inference or using high-d KDEs
  - Public codes and data products (e.g. RIFT GW likelihood)
Asymmetric binaries: Exceptional event as example

- Asymmetric binaries: **much** tighter constraints

**GW190814 (Abbott et al 2020)**
Why can we learn so much?

- NS in asymmetric binaries: **much** tighter constraints
- Precession (and higher modes) can break degeneracies

12d MCMC vs 7d Fisher
ROS et al 2014 (PRD 89 102005)
Approximate precessing kinematics

\[ \partial_t \mathbf{X} = \Omega_{\mathbf{X}} \times \mathbf{X}, \quad \mathbf{X} = \mathbf{L}, \mathbf{S}_1, \mathbf{S}_2 \]

- Example: one spin

\[ \frac{d\hat{\mathbf{L}}}{dt} \simeq \frac{\mathbf{J}}{r^3} \left( 2 + \frac{3m_2}{2m_1} \right) \times \hat{\mathbf{L}} \]

\[ |\mathbf{J}| = |\mathbf{L} + \mathbf{S}| \]

- Extend known single-spin precession solutions
  - \( \beta(v) \): set by |L| and (conserved) L.S
  - \( \alpha \): precession phase

\[ = \int \Omega_p \frac{dt}{dv} dv \]

: **analytic approximations** exist

Apostolatos et al 1994; Lundgren and ROS 2013
Sample precessing geometry: BH-NS

\[ \begin{align*}
\hat{J} &\quad \hat{N} \\
\hat{L} &\quad \hat{x}_N \\
\hat{x} &\quad \hat{y}_N
\end{align*} \]

\[ \begin{align*}
\beta &\quad \theta_{JN} \\
\alpha &\quad \psi_J
\end{align*} \]

\[ \begin{align*}
\cos \alpha &\quad \cos \beta \\
\cos \psi_J &\quad \cos \theta_{JN}
\end{align*} \]
Simple approximate (intrinsic) Fisher matrix

\[ \rho^2_{2ms} \equiv \left| -2Y_{2m}(\theta_J^N) d^2_{m,2s}(\beta) \right|^2 \int_0^\infty df \frac{4(\pi M_c^2)^2}{S_h(f) 3 d_L^2} (\pi M_c f)^{-7/3} \]

\[ \hat{\Gamma}_{ab}^{(ms)} = \frac{\int_0^\infty df \frac{df}{S_h(f)} (\pi M_c f)^{-7/3} \partial_a (\Psi_2 - 2\zeta - ms\alpha) \partial_b (\Psi_2 - 2\zeta - ms\alpha)}{\int_0^\infty df \frac{df}{S_h(f)} (\pi M_c f)^{-7/3}} \]

• Good:
  • Easy to calculate
  • Similar to nonprecessing (weighted average)
  • Intuition about separating parameters

• “Bad”
  • Ansatz / approximation
  • At best, retains all degeneracies of full problem (phases, …)
Precession-induced modulation

- Chirp rate, precession rate set limits
  - More cycles -> more accuracy
- Precession enables measurements
  - Spin-orbit misalignment
  - Mass ratio \( \times 3 \) better

\[
\begin{align*}
\text{constant } \beta & \quad \text{constant } \Omega_p
\end{align*}
\]
Simple approximate (intrinsic) Fisher matrix

\[ \rho_{2ms}^2 \equiv \left| -2 Y_{2m}(\theta_{JN}) d_{m,2s}^2(\beta) \right|^2 \int_0^\infty \frac{df}{S_h(f)} \frac{4(\pi M_c f)^2}{3d_L^2} (\pi M_c f)^{-7/3} \]

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- **Amplitude**
- **Angular dependence**
- **Phase**
Do you want to use GW observations for EOS constraints and/or population inference? But…

• Don’t want to reinvent GW inference or keep up with the latest GW waveform models yourself?

• Worried about introducing approximations (KDEs, fixed chirp masses, …) to make your calculation tractable?

• Don’t want to invent your own population inference code to combine many events?

• PopModels & RIFT do this all for you!

• Extensively used in prior work (e.g., O1/O2 BBH by LVC)

• This project: proof of concept for fast, easy multi-population work

• Code associated with this project ported to main repo soon!

https://git.ligo.org/daniel.wysocki/bayesian-parametric-population-models
Combine everything

- Joint distribution: **mass, mass ratio, spin, tides**
  - Needed/valuable due to correlated measurement uncertainties
Combine everything

- Joint distribution: mass, mass ratio, spin, tides
  - Needed/valuable due to correlated measurement uncertainties
...and some BNS astrophysics too

- O3: Expect O(1) event has measurable/nonzero spin
  - Some galactic merging binary NS have roughly $\chi \approx 0.01$
    \[ \chi = \frac{S}{M^2} \approx 0.5 (f \text{ ms}) \]

- BNS mass/spin distribution has lots of value itself!
  - constrain NS spin down models [Zhu et al 1711.09226]
Nuclear equation of state: Many observations

- Joint distribution: **mass, mass ratio, spin, tides**
  - Needed/valuable due to correlated measurement uncertainties

- Synthetic data example (APR4, narrow BNS masses,7 **loud** events)
  - Same code can immediately exploit new constraints (EM BNS mergers, X-ray,...)

Wysocki, ROS et al 2018
Wysocki, ROS et al in prep
Nuclear equation of state

• Other news:
  • Models: Improved parameterizations/nonparametric, nuclear theory inputs, etc
    • e.g., Essick 2018 (1811.12529), Reddy 2019, …
  • Complementary data:
    • NICER (radius measurements from X-ray bursts) soon
    • New NS with high mass \((2.17 \pm 0.1)\) (PSR J0740+6620, Cromartie et al 2019)
  • Modeling work:
    • Debate on lower bound on tides set by large ejecta (Radice 2018-> Kiuchi et al 2019)
  • Comparisons of EOS to GW observations:
    • Updated analysis from LIGO pending, comparing different EOS
Beyond the mass distribution: Power of spin

- Misalignments trace key kinematic effects (kicks or dynamics)
  - “Single spin” (e.g., unequal mass or BH-NS binary):
    - Key misalignment is \( \sim \text{conserved} \) since past infinity.
    - Easy to interpret for astrophysics
    - Very many GW and precession cycles possible
    - Strong precession requires high mass ratio and BH spin
  - “Two spin” (e.g., comparable mass):
    - both spins accessible

![Graph showing contours of constant alignment angle as a function of BH spin and orbital angular momenta.](image-url)