Incorporating Radiation into Relativistic Fluid Codes

Josh Dolence
Computational Physics & Methods
Center for Theoretical Astrophysics
Los Alamos National Laboratory

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Collaborators: Ben Ryan (LANL), Jonah Miller (LANL), Charles Gammie (UIUC)

Radiation in Relativistic Problems

- Relativistic environments have an enormous range of thermodynamic conditions
- Low-density environments often cannot radiate efficiently, leading to problems that are essentially non-radiative (dynamically); e.g. LLAGN
- High-density environments can have non-negligible optical depths even for neutrinos, leading to significant energy transfer and lepton number evolution
- Radiation can dominate thermal and/or dynamical evolution, and play an essential role in chemical evolution (or it might not matter at all)
- The key to successfully incorporating radiation into relativistic fluid codes is to understand the nature of the interaction *in your application* and proceed accordingly – no one size fits all solution

What is radiation transport?

- The evolution of a population of photons/neutrinos as they get created/destroyed, propagate, and scatter
- "Simple":

$$\frac{Df_i}{d\lambda} = \mathcal{C} \quad \text{Liouville's Theorem}$$

$$i \in \{\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau\}$$

$$f = f(x^\mu, \mathbf{\Omega}, \varepsilon) \equiv \text{Phase space density}$$

$$\lambda \equiv \text{Affine parameter}$$

- Major challenges continue to occupy careers/journals/etc.
  1. Evolution of a 6-D problem (really 6 6-D problems for neutrinos!). **Expensive!**
  2. The "convective derivative" in phase space can be nontrivial, and quite horrible in all but flat space Cartesian coordinates. Choice of frame?

$$\frac{D}{d\lambda} = k^\alpha \left(\frac{\partial}{\partial x^\alpha} - \Gamma^\mu_{\alpha\beta} k^\beta \frac{\partial}{\partial k^\mu} \right)$$

$$\Rightarrow \frac{\partial}{\partial t} + \hat{n} \cdot \nabla \quad \text{(Flat space, Cartesian coordinates)}$$

  3. Collisionless to Collisional

Evaluating the Convective Derivative

Choose a frame any frame…

- Laboratory frame is the frame of choice for evaluating the transport operator
- One fair comparison: 1-D spherical special relativistic transport

$$\frac{1}{c}\frac{\partial I}{\partial t} + \mu \frac{\partial I}{\partial r} + \frac{1-\mu^2}{r}\frac{\partial I}{\partial \mu} = \mathrm{RHS}$$

Lab frame

$$\begin{aligned}
&\frac{\gamma}{c}\left(1+\beta\mu_0\right)\frac{\partial I_0}{\partial t} + \gamma\left(\mu_0+\beta\right)\frac{\partial I_0}{\partial r} \\
&+ \frac{\partial}{\partial \mu_0}\left\{\gamma\left(1-\mu_0^2\right)\left[\frac{1+\beta\mu_0}{r} - \gamma^2\left(\mu_0+\beta\right)\frac{\partial \beta}{\partial r} - \frac{\gamma^2}{c}\left(1+\beta\mu_0\right)\frac{\partial \beta}{\partial t}\right] I_0\right\} \\
&- \frac{\partial}{\partial \nu_0}\left\{\gamma\nu_0\left[\frac{\beta\left(1-\mu_0^2\right)}{r} + \gamma^2\mu_0\left(\mu_0+\beta\right)\frac{\partial \beta}{\partial r} + \frac{\gamma^2}{c}\mu_0\left(1+\beta\mu_0\right)\frac{\partial \beta}{\partial t}\right] I_0\right\} \\
&+ \gamma\left\{\frac{2\mu_0+\beta\left(3-\mu_0^2\right)}{r} + \gamma^2\left(1+\mu_0^2+2\beta\mu_0\right)\frac{\partial \beta}{\partial r} + \frac{\gamma^2}{c}\left[2\mu_0+\beta\left(1+\mu_0^2\right)\right]\frac{\partial \beta}{\partial t}\right\} I_0 = \mathrm{RHS}
\end{aligned}$$

Comoving frame

- So why would anyone ever choose the comoving frame?

What about the RHS?

- But the comoving frame is the natural choice for interactions. Why?
- Natural frame to describe the statistics of interacting electrons, nucleons, etc; Think tables!
- Lorentz invariants are your friend $f \propto \frac{I_\nu}{\nu^3}$

$$\frac{D}{d\lambda}\left(\frac{I_\nu}{\nu^3}\right) = \left(\frac{j_\nu}{\nu^2}\right) - (\nu\kappa_\nu)\left(\frac{I_\nu}{\nu^3}\right)$$

- How does this help us?

$$\nu_0 \propto -p^\mu u_\mu$$

- So just dot the neutrino four-momentum with the material four velocity to get the comoving frame energy, evaluate coefficients, and form the invariants

$$\frac{j_\nu}{\nu^2} = \frac{j_{\nu_0}}{\nu_0^2}$$

$$\nu\kappa_\nu = \nu_0\kappa_{\nu_0}$$

But there's more…

- Developing a scheme that appropriately handles both the collisionless and collisional limits is *not* trivial
- In the transport literature, this is often discussed in the context of a numerical method's asymptotic behavior
- In the optically thick regime, the distribution must approach
  1. Equilibrium; isotropy ***in the comoving frame***)
  2. Static/Dynamic Diffusion; Fick's law ***in the comoving frame***
- In the optically thin regime, F => cE ***in the lab frame***
- Many schemes have trouble in one or both limits and almost all rely on O(v/c) expansions
  - Beware O(v/c) expansions
- Bottom line: *if* we must expand the Liouville operator to solve a discrete version of the 6-D transport problem on a mesh, the resulting scheme will be expensive and complicated

Monte Carlo Methods

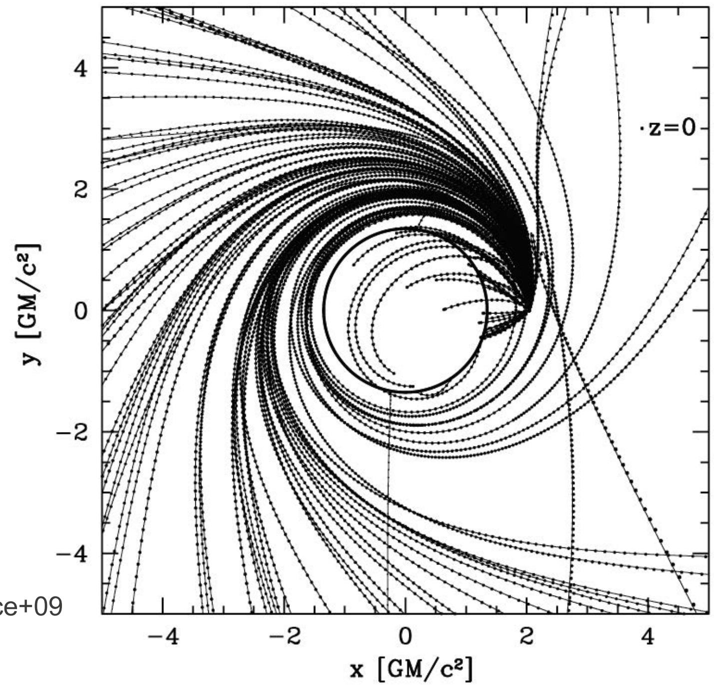
- A large class of statistical methods used to solve transport (and other) problems
- Convergence rate is $O(N^{-1/2})$, independent of dimension
- Basic method involves evolving a collection of sample particles that each represent a large number of radiation particles
- Most popular method referred to as Implicit Monte Carlo (IMC); Fleck & Cummings (1971)
  - Many others exist: symbolic implicit Monte Carlo, exponentially convergent Monte Carlo, difference Monte Carlo, ….
- Monte Carlo methods are often "close to the physics", which is often advantageous for physicist developers. Contrast this with schemes that rely on large solver libraries.

Monte Carlo Methods

- **Advantages**
  - The transport operator becomes extremely simple – every sample has a well-defined x^μ and k^μ, and they move along geodesics. **Especially important in relativistic applications.**

$$\frac{dx^\mu}{d\lambda} = k^\mu$$
$$\frac{dk^\mu}{d\lambda} = -\Gamma^\mu_{\alpha\beta} k^\alpha k^\beta$$

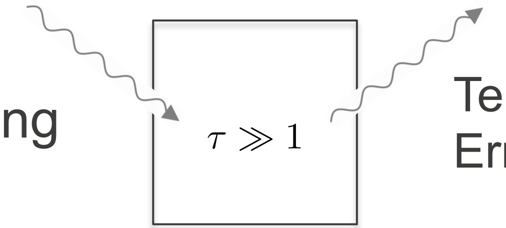
  - Simultaneously, use of Lorentz invariance and/or straightforward boosting enables the treatment of complex interactions
- **Disadvantages**
  - Noise
  - High optical depths
  - Stiffness
  - Significant radiation pressure
  - Particles often an afterthought



Dolence+09

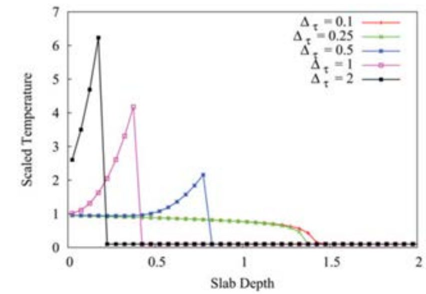
Monte Carlo Skeletons

- Basic scheme is *not* asymptotic preserving



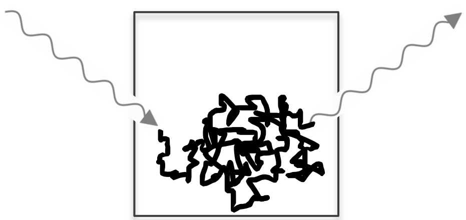
Teleportation Error

- Admits unphysical solutions, e.g. can violate maximum principle



Wollaber 2013

- Odd moments (like the flux and radiation force) suffer catastrophically in the nearly isotropic limit. "Effective" fraction of samples in flux calculation is $\sim \frac{F}{cE} \ll 1$ in the diffusion regime
- Efficiency drops precipitously in optically thick regions



Monte Carlo Strengths

- Relatively straightforward method that delivers full transport solutions without undesirable approximations (diffusion, M1, etc.)
- It is *not* always required that we have a highly accurate representation of the radiation field at all points in space and time
  - Can accommodate adaptivity
  - Effective number of samples can be much larger than samples/cell if relevant timescales are long compared to light crossing time of a cell
  - Character of error is fundamentally different than in deterministic methods
- Free streaming and other highly anisotropic phenomena (e.g. high Lorentz factor flows) are handled naturally
- Overall cost *can* be much lower than for deterministic methods (but *can* also be much higher – depends on the problem)

Advanced Topics

- Variance reduction techniques
  - Methods that reduce the variance of Monte Carlo estimates **without modifying the expectation value**
  - Particle splitting, Russian Roulette, biasing
- Techniques to improve efficiency in diffusive regimes
  - Discrete diffusion Monte Carlo (Densmore+07, Abdikamalov+12)
  - Random walk acceleration (Richers+17)
- Stiff inelastic scattering (e.g. Compton)

- Enormous literature on Monte Carlo methods for transport that goes back ~70 years

Existence Proofs – Full GR Radiation MHD

- Both based on explicit Monte Carlo, not IMC
- `ebhlight`: Ryan, Dolence, & Gammie (2015)
  - Open source (github.com/AFD-Illinois/ebhlight)
  - GRMHD is a HARM (Gammie+03) derivative
  - Transport derives from `grmonty` (Dolence+09) (github.com/AFD-Illinois/igrmonty)
  - Includes two-temperature physics, synchrotron, bremsstrahlung, Compton
  - Applied to systems like M87
- `νbhlight`: Miller, Ryan, & Dolence (2019)
  - Open source (github.com/lanl/nubhlight)
  - Derives from `ebhlight`, adding general (usually tabulated) EOS, multi-species neutrino transport, and associated lepton number evolution
  - Applied to accretion in post merger environments and in collapars

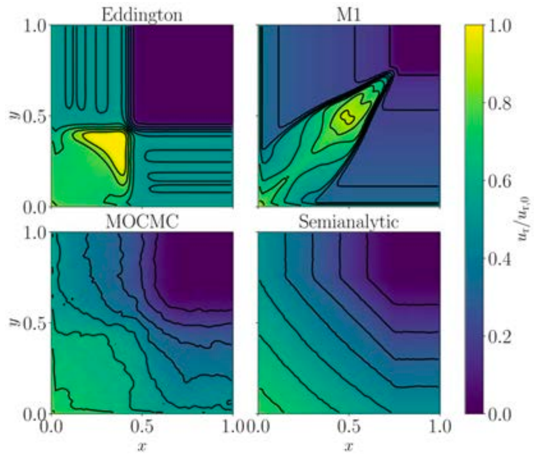
Overcoming Monte Carlo's Pitfalls

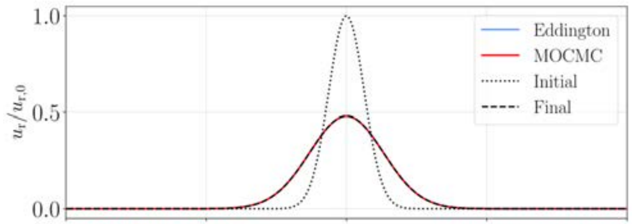
- MOCMC – Method of Characteristics Moment Closure (Ryan & Dolence 2020)
- Gray moment system introduced which mediates coupling between gas and radiation
- Retains particle based discretization of phase space, including geodesic integration
- Drops stochastic sampling of interactions in favor of deterministic (and implicit) coupling
- Reconstructions of phase space distribution in each cell lead to higher-order convergence than Monte Carlo; used to close the moment system and provide appropriately averaged opacities
- Each MOCMC sample carries a complete spectrum, *dramatically* lowering the cost of the transport operator

MOCMC

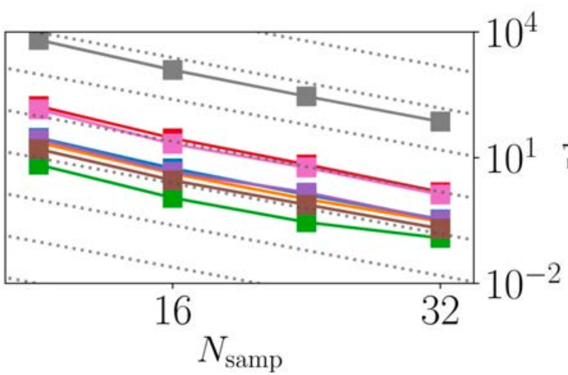
- Naturally handles relativistic applications, just like Monte Carlo
- Implicit nonlinear coupling guarantees stability
- Accurate and performant for wide-range of optical depths and radiation-to-gas pressures, from $\ll 1$ to $\gg 1$

Free streaming

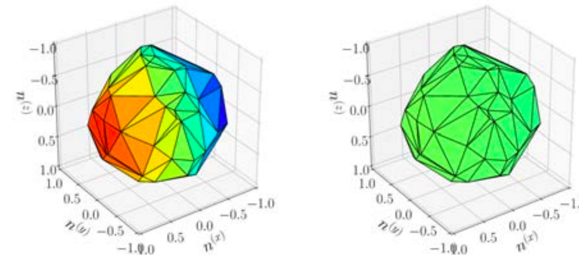




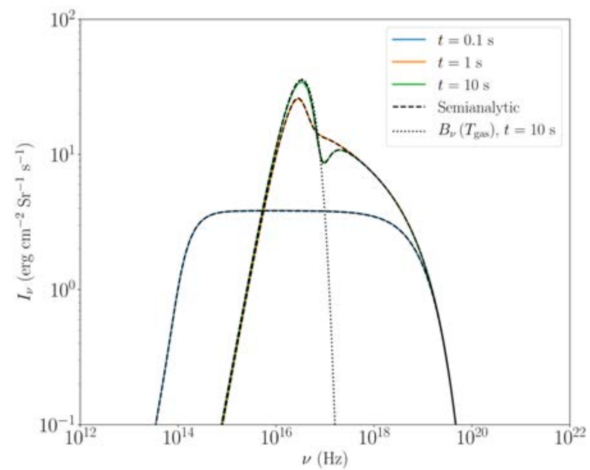
Preserves diffusion limit



2nd order convergence on radiation-modified fast magnetosonic modes with P_r=10 P_g



Isotropization via Comptonization



Full spectral evolution

Conclusions

- Radiation can play many roles – understand its importance in your application(s) before adopting a method
- Direct discretizations of 6-D phase space are complicated and expensive
- Monte Carlo offers many advantages in relativistic applications, but may struggle if optical depths are too large or radiation forces are too important
- MOCMC offers many attractive features, but remains to be battle tested in the wild

- Check out the existing open source GR Radiation MHD codes `bhlight` and `νbhlight`!
- Also might be interested in Parthenon (github.com/lanl/parthenon)