Nuclear equations of state: underlying physics and use in fluid codes

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Physics in the accretion disk: why do we need neutrinos+realistic EOS?

Chen & Beloborodov (2016)
Di Matteo et al. (2002)
Narayan et al. (2001)
Neutrino reactions

Charged beta-process

\[ e^- + p \rightarrow n + \nu_e \]
\[ e^+ + n \rightarrow p + \bar{\nu}_e \]

Electron-positron pair annihilation

\[ e^- + e^+ \rightarrow \nu_e + \bar{\nu}_e \]
\[ e^- + e^+ \rightarrow \nu_x + \bar{\nu}_x \]

Plasmon decay

\[ \gamma \rightarrow \nu_e + \bar{\nu}_e \]
\[ \gamma \rightarrow \nu_x + \bar{\nu}_x \]

Absorption (opacity source)

\[ \nu_e + n \rightarrow p + e^- \]
\[ \bar{\nu}_e + p \rightarrow n + e^+ \]
Accretion disk

Gravitationally unstable (Toomre Q)

Chen & Beloborodov (2016)
Accretion disk

Neutrino transport (or leakage scheme)

Chen & Beloborodov (2016)
Nucleosynthesis in accretion disks

The final composition is still uncertain

e.g. Wu et al. (2016), Siegel & Metzger (2018), Fernandez et al. (2018), Foucart et al. (2018), Miller et al. (2019)
Working group 2 objectives

- Interpolation of tables from stellarcollapse.org
- Naturally takes care of alpha-particles and neutrino pressure terms
- Calculate the opacity averaged in energy of each species of neutrino given the temperature and density
- Calculate the optical depth
- Add source terms into energy and number equation

Realistic EOS in HARM3d

Neutrino leakage scheme in HARM3d
EOS interpolation


- Added it to the conserved to primitive variable solver that solves a set of equations using a Newton-Raphson method to get the primitive variables from the conserved ones.

- Added the temperature as a primitive variable.

- The characteristic velocity also calls the EOS to get the sound speed.

- The primitive variable energy now depends on the primitive variables temperature, electron fraction and density. There is an EOS call to link them.
EOS interpolation: conserved to primitive variables solver
see Siegel et al. (2018)

Routine 1: 2d method- Noble et al. (2006)

Equations to solve
\[
\tilde{Q}^2 = v^2 (B + W)^2 - \frac{(Q_\mu B^\mu)^2 (B^2 + 2W)}{W^2}
\]
\[
Q_\mu n^\mu = -\frac{B^2}{2} (1 + v^2) + \frac{(Q_\mu B^\mu)^2}{2W^2} - W + P(\rho, Y_e, T)
\]

Independent variables:
\[v, W\]

Conserved variables:
\[Q_\mu = \alpha T^t_\mu, \quad B^i = \alpha B^i\]
EOS interpolation: conserved to primitive variables solver see Siegel et al. (2018)

**Routine 2: 2d method-Noble et al. (2006)**
+ safe guess-Cerdá-Durán et al. (2008)

\[ \Gamma_{\text{guess}} = 10000 \quad \text{Adds robustness} \]
\[ W = Q_\mu n^\mu + P_{\text{max}} - \frac{B^2}{2} \]

**Routine 3: 2d method-Noble et al. (2006)**
+ dog leg method-Press et al. (1992)

Dog leg method allows for a more robust convergence if the solution is away from the initial guesses
EOS interpolation: conserved to primitive variables solver see Siegel et al. (2018)

Routine 4: 3d method- Cerdá-Durán et al. (2008), Siegel et al. (2018)

Equations to solve

\[
\tilde{Q}^2 = \left(1 - \frac{1}{\Gamma^2}\right)(B + W)^2 - \frac{(Q_{\mu}B^{\mu})^2(B^2 + 2W)}{W^2}
\]

\[
Q_{\mu}n^{\mu} = -\frac{B^2}{2} \left(2 - \frac{1}{\Gamma^2}\right) + \frac{(Q_{\mu}B^{\mu})^2}{2W^2} + W + P(\rho, Y_e, T)
\]

\[
\epsilon = \epsilon(\rho, Y_e, T)
\]

Independent variables:

\[W, T, \Gamma\]
Testing EOS interpolation

• To test the EOS tables, we can use the relative error after the conversion from conserved variables to primitive variables.

• Here is the relative error comparing several routines. The density is in cgs, the temperature in K.

Based on Siegel et al. (2018)
Driver from O'Connor & Ott (2010),
Schneider et al. (2017)
Subtleties with the EOS: Atmosphere

Isentropic Fishbone Moncrief disk

Atmosphere with same entropy as the disk and $h=1$

Atmosphere with higher entropy as the disk and $h=1$

In code units
Subtleties with the EOS: Atmosphere

Isentropic Fishbone Moncrief disk with outside enthalpy similar to minimum enthalpy in the disk
Leakage scheme

**Charged beta-process**

\[ e^- + p \rightarrow n + \nu_e \]
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**Plasmon decay**

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**Absorption (opacity source)**

\[ \nu_e + n \rightarrow p + e^- \]
\[ \bar{\nu}_e + p \rightarrow n + e^+ \]

**Scattering with free nucleons**

Based on Ruffert et al. (1996)
Galeazzi et al. (2013)
Bruenn (1985) and other papers
Leakage scheme

Source terms

\[ \nabla_\mu T^{\mu \nu} = Q u^\nu \]
Heating/cooling rate

\[ \nabla_\mu (n_e u^\mu) = R \]
Absorption/emission rate

The effective rates are an interpolation between the optically thin and thick regime

\[ R_{\nu}^{\text{eff}} = \frac{R_{\nu}}{1 + \frac{t_{\text{diff}}}{t_{\text{emission},R}}} \]

\[ Q_{\nu}^{\text{eff}} = \frac{Q_{\nu}}{1 + \frac{t_{\text{diff}}}{t_{\text{emission},Q}}} \]

Based on Ruffert et al. (1996), Galeazzi et al. (2013), with modifications from Rosswog & Liebendörfer (2003), Siegel & Metzger (2018), O'Connor & Ott (2010)

\[ t_{\text{diff}} = \frac{6\tau^2}{c \kappa_{\nu_i}} \]

\[ t_{\text{emission},R} = \frac{R_{\nu_i}}{n_{\nu_i}} \]

\[ t_{\text{emission},Q} = \frac{Q_{\nu_i}}{\varepsilon_{\nu_i}} \]
Leakage scheme

If the diffusion timescale is large (opaque region):

\[ R_{\nu}^{\text{eff}} = \frac{n_{\nu}}{t_{\text{diff}}} \]
\[ Q_{\nu}^{\text{eff}} = \frac{\epsilon_{\nu}}{t_{\text{diff}}} \]

\[ \kappa(\bar{\nu}_e) = \kappa_s(\bar{\nu}_e, n) + \kappa_s(\bar{\nu}_e, p) + \kappa_a(\bar{\nu}_e, p) \]
\[ \kappa(\nu_e) = \kappa_s(\nu_e, n) + \kappa_s(\nu_e, p) + \kappa_a(\nu_e, n) \]

In transparent region:

\[ R_{\nu}^{\text{eff}} = R_{\nu} \]

\[ R_{\nu} = R_{\beta-\text{charged}} + R_{\text{plasmon decay}} + R_{e^- e^+} \]

(same for Q)

Based on Ruffert et al. (1996)
Galeazzi et al. (2013)
Leakage scheme testing

Evolution of isotropic, optically thin, constant density gas

\[ \partial_t u = Q \]
\[ \partial_t Y_e = R/\rho \]

Beta process for electron antineutrino

Ryan et al. (2015)
Miller et al. (2019)
Optical depth

$$\tau = \int_{s_1}^{s_2} \kappa ds$$

Neilsen et al. (2014), Siegel & Metzger (2018)

$$\min(\tau_{\nu,\text{neigh}} + \bar{\kappa}_\nu (\gamma_{ab} dx^a dx^b)^{1/2})$$

Comes into the calculation of the diffusion time
Leakage scheme: optical depth

\[ \tau = \int_{s_1}^{s_2} \kappa ds \]

Testing a sphere of constant density and temperature

\[ \min(\tau_{\nu,\text{neigh}} + \bar{\kappa}_\nu (\gamma_{ab} dx^a dx^b)^{1/2}) \]

Neilsen et al. (2014), Siegel & Metzger (2018)

Optical depth to electron neutrinos
Leakage scheme: optical depth

\[
\tau = \int_{s_1}^{s_2} \kappa ds
\]

Neilsen et al. (2014), Siegel & Metzger (2018)

\[
\min(\tau_{\nu,\text{neigh}} + \bar{\kappa}_{\nu}(\gamma_{ab} dx^a dx^b)^{1/2})
\]

With our convergence criterion:

Testing a sphere of constant density and temperature

Optical depth to electron neutrinos
Conclusions and future work

• Tabulated EOS and neutrino leakage scheme are ready to work!

• Future work: We need to understand more the initial conditions in the atmosphere.

• Future work: add more robust utoprim routines

• Future work: work on using skynet to do the nucleosynthesis.
¡Gracias!