Nuclear equations of state: underlying physics and use in fluid codes

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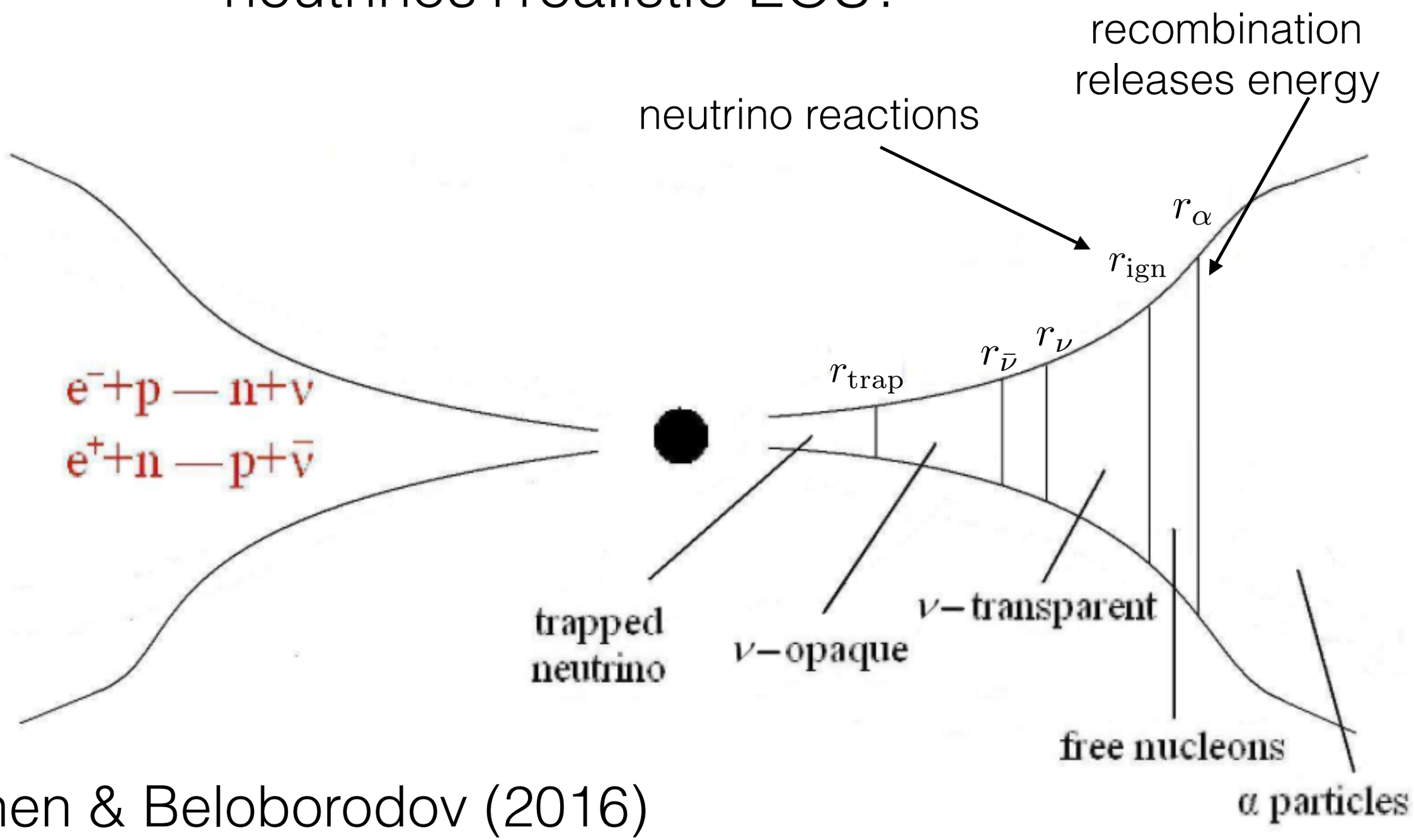
NASA's Conceptual Imaging Lab

Physics in the accretion disk: why do we need neutrinos+realistic EOS?



Chen & Beloborodov (2016)

Di Matteo et al. (2002)

Narayan et al. (2001)

Neutrino reactions

Charged beta-process

$$e^- + p \rightarrow n + \nu_e$$

$$e^+ + n \rightarrow p + \bar{\nu}_e$$

Plasmon decay

$$\gamma \rightarrow \nu_e + \bar{\nu}_e$$

$$\gamma \rightarrow \nu_x + \bar{\nu}_x$$

Electron-positron pair annihilation

$$e^- + e^+ \rightarrow \nu_e + \bar{\nu}_e$$

$$e^- + e^+ \rightarrow \nu_x + \bar{\nu}_x$$

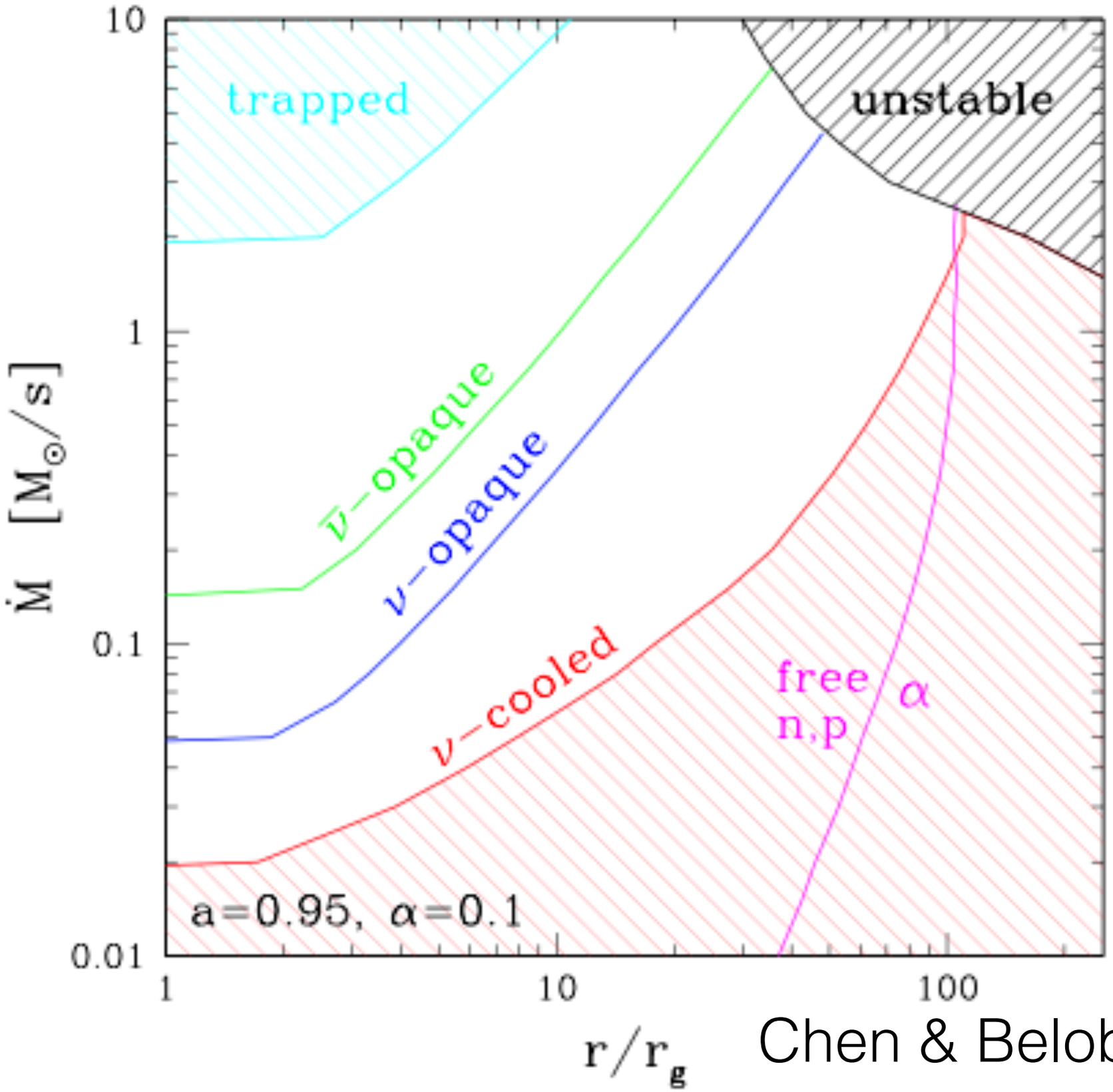
Absorption (opacity source)

$$\nu_e + n \rightarrow p + e^-$$

$$\bar{\nu}_e + p \rightarrow n + e^+$$

Accretion disk

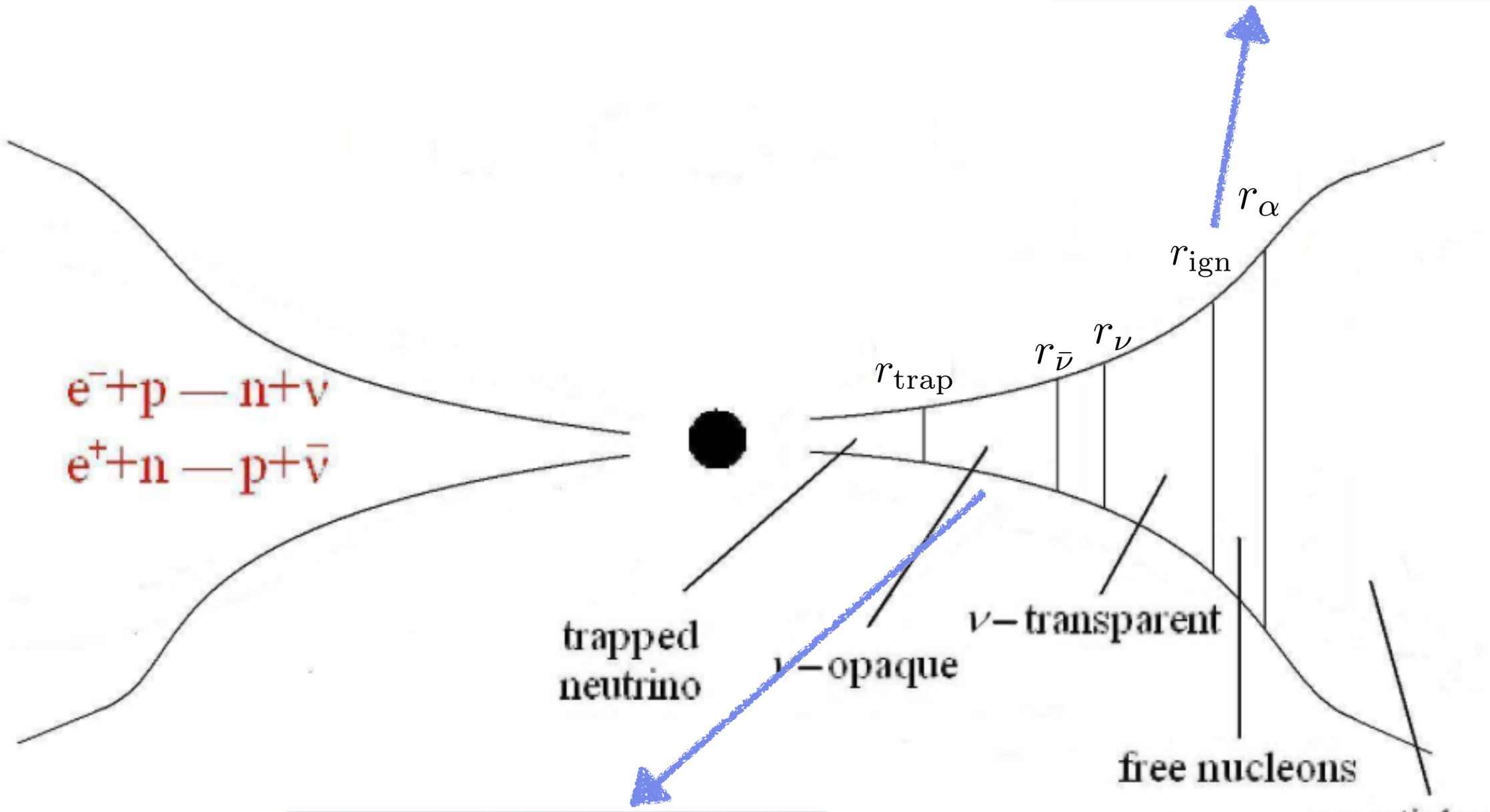
Gravitationally unstable (Toomre Q)



Chen & Beloborodov (2016)

Accretion disk

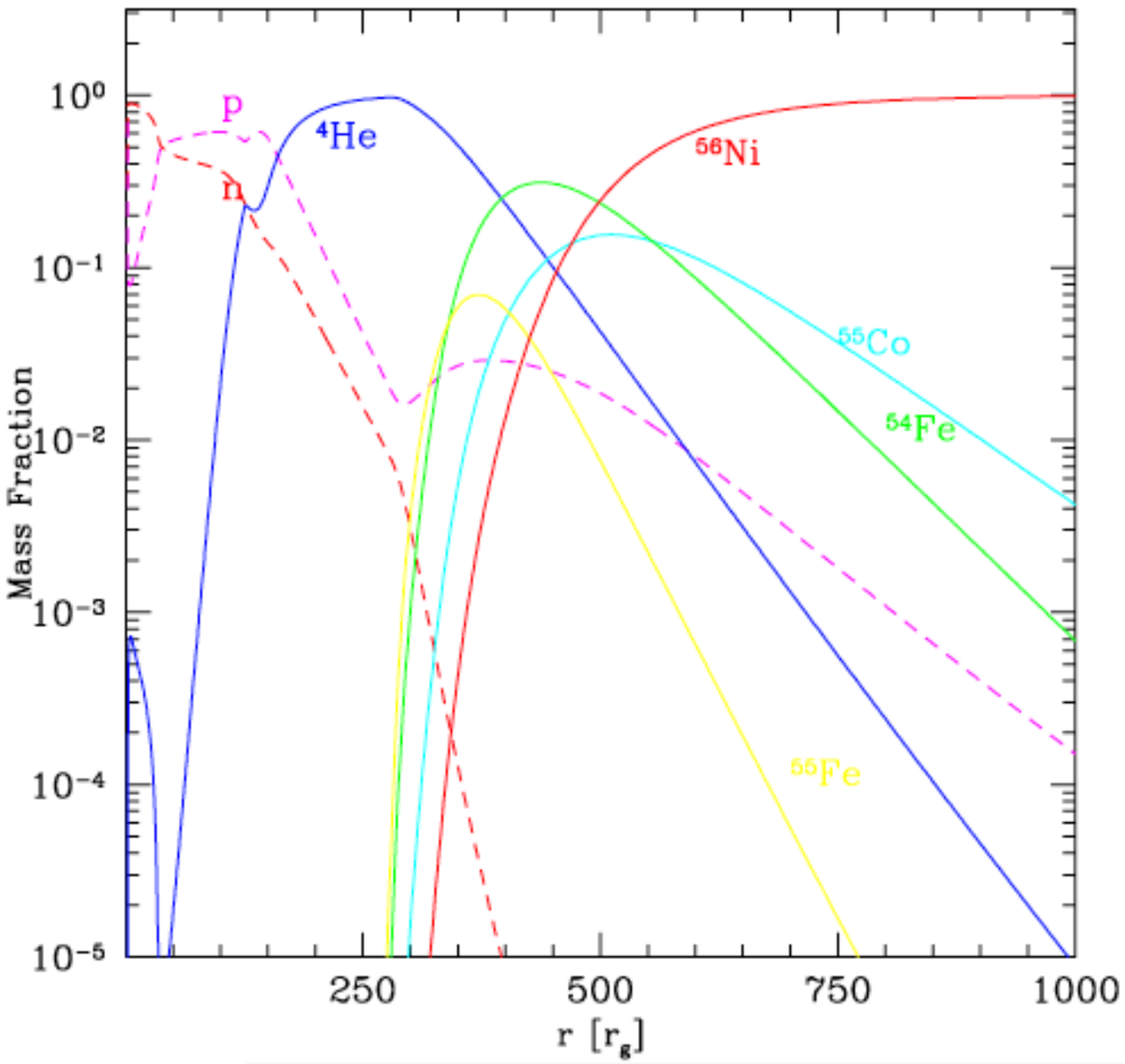
Realistic EOS

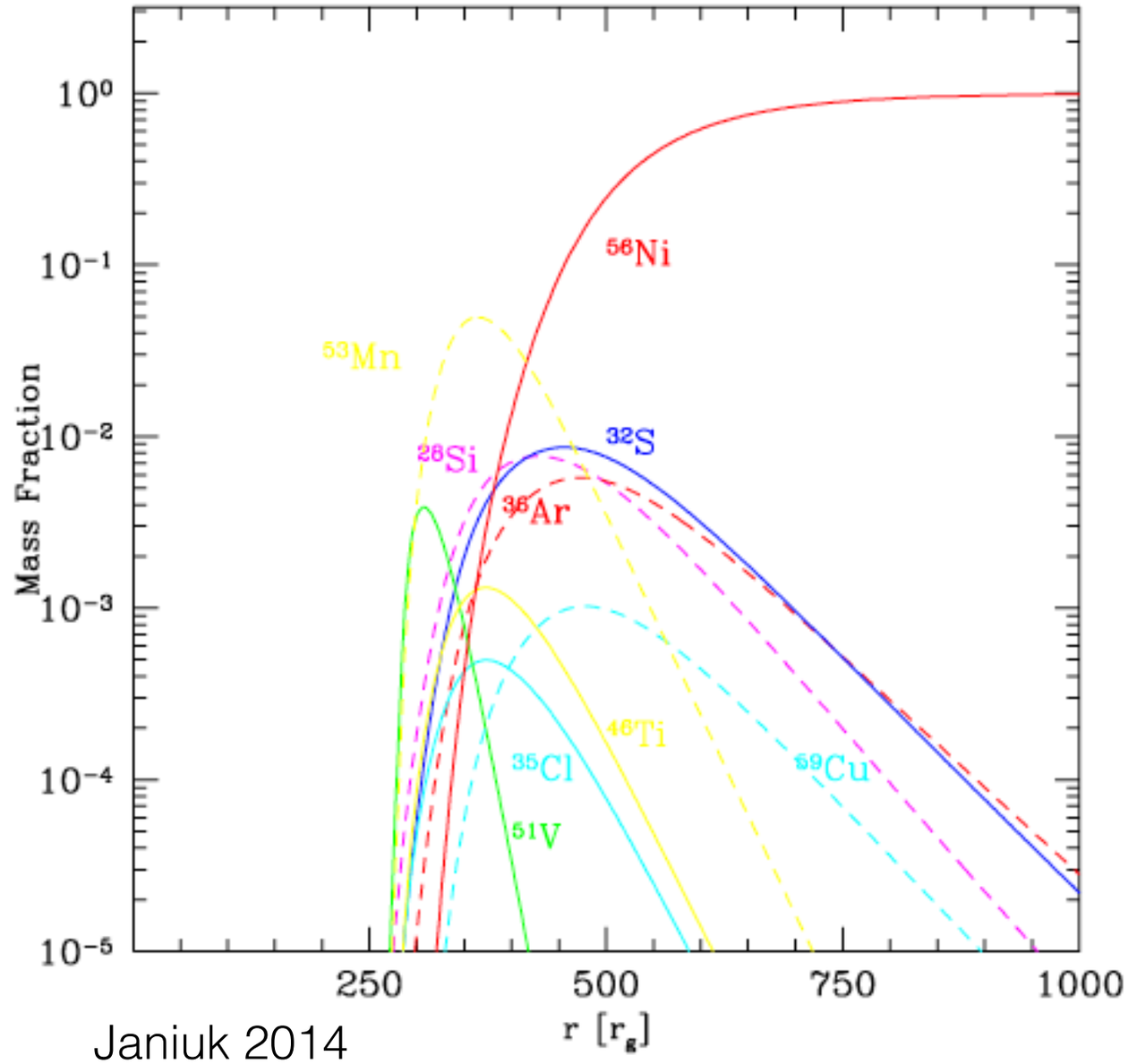


Neutrino transport
(or leakage scheme)

Chen & Beloborodov (2016)

Nucleosynthesis in accretion disks



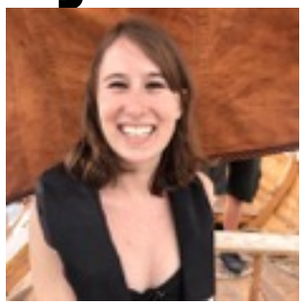


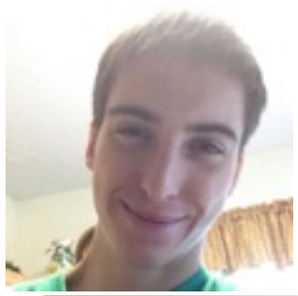
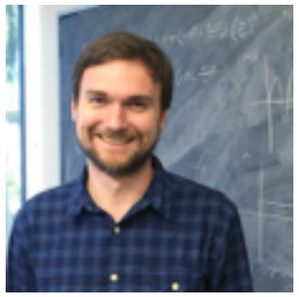
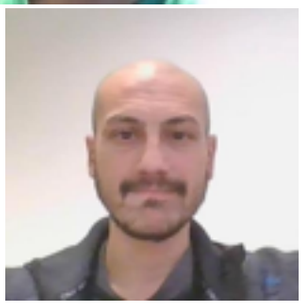
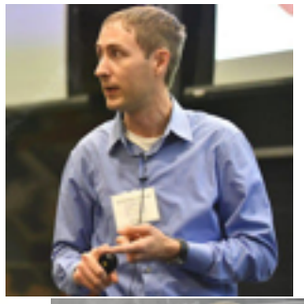
Janiuk 2014
Janiuk et al. 2013
Janiuk et al. 2007

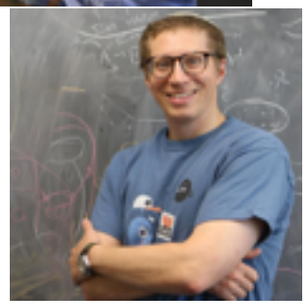
The final composition is still uncertain

e.g. Wu et al. (2016), Siegel & Metzger (2018), Fernandez et al. (2018), Foucart et al. (2018), Miller et al. (2019)

Working group 2 objectives






Realistic EOS in HARM3d

- Interpolation of tables from stellarcollapse.org
- Naturally takes care of alpha-particles and neutrino pressure terms

Neutrino leakage scheme in HARM3d

- Calculate the opacity averaged in energy of each species of neutrino given the temperature and density
- Calculate the optical depth
- Add source terms into energy and number equation

EOS interpolation

- Interpolation from O'Connor & Ott (2010), Schneider et al. (2017). Based on Siegel et al. (2018).
- Added it to the conserved to primitive variable solver that solves a set of equations using a Newton-Raphson method to get the primitive variables from the conserved ones.
- Added the temperature as a primitive variable.
- The characteristic velocity also calls the EOS to get the sound speed.
- The primitive variable energy now depends on the primitive variables temperature, electron fraction and density. There is an EOS call to link them.

EOS interpolation: conserved to primitive variables solver see Siegel et al. (2018)

Routine 1: 2d method- Noble et al. (2006)

$$\tilde{Q}^\mu = j^\mu{}_\nu Q^\nu$$
$$j_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$$
$$W = \rho h \Gamma^2$$
$$h = 1 + P + \epsilon$$

Equations to solve

$$\tilde{Q}^2 = v^2(\mathcal{B} + W)^2 - \frac{(Q_\mu \mathcal{B}^\mu)^2(\mathcal{B}^2 + 2W)}{W^2}$$

$$Q_\mu n^\mu = -\frac{\mathcal{B}^2}{2}(1+v^2) + \frac{(Q_\mu \mathcal{B}^\mu)^2}{2W^2} - W + P(\rho, Y_e, T)$$

Independent variables:

v

W

Conserved variables:

$$Q_\mu = \alpha T^t{}_\mu$$
$$\mathcal{B}^i = \alpha B^i$$

EOS interpolation: conserved to primitive variables solver see Siegel et al. (2018)

Routine 2: 2d method-Noble et al. (2006) +safe guess-Cerdá-Durán et al. (2008)

$$\Gamma_{\rm guess} = 10000$$

$$W = Q_\mu n^\mu + P_{\rm max} - \frac{\mathcal{B}^2}{2}$$

Adds robustness

Routine 3: 2d method-Noble et al. (2006) +dog leg method-Press et al. (1992)

Dog leg method allows for a more robust convergence if the solution is away from the initial guesses

EOS interpolation: conserved to primitive variables solver see Siegel et al. (2018)

Routine 4: 3d method- Cerdá-Durán et al. (2008), Siegel et al. (2018)

Equations to solve

$$\tilde{Q}^2 = \left(1 - \frac{1}{\Gamma^2}\right)(\mathcal{B} + W)^2 - \frac{(Q_\mu \mathcal{B}^\mu)^2(\mathcal{B}^2 + 2W)}{W^2}$$

$$Q_\mu n^\mu = -\frac{\mathcal{B}^2}{2}(2 - \frac{1}{\Gamma^2}) + \frac{(Q_\mu \mathcal{B}^\mu)^2}{2W^2} - W + P(\rho, Y_e, T)$$

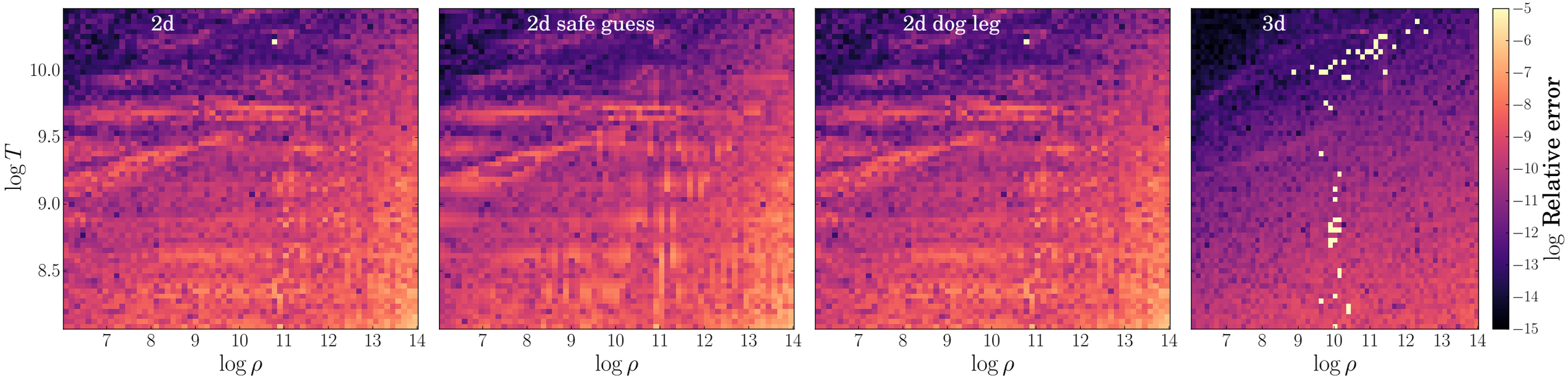
$$\epsilon = \epsilon(\rho, Y_e, T)$$

Independent variables: W, T, Γ

Testing EOS interpolation

- To test the EOS tables, we can use the relative error after the conversion from conserved variables to primitive variables.

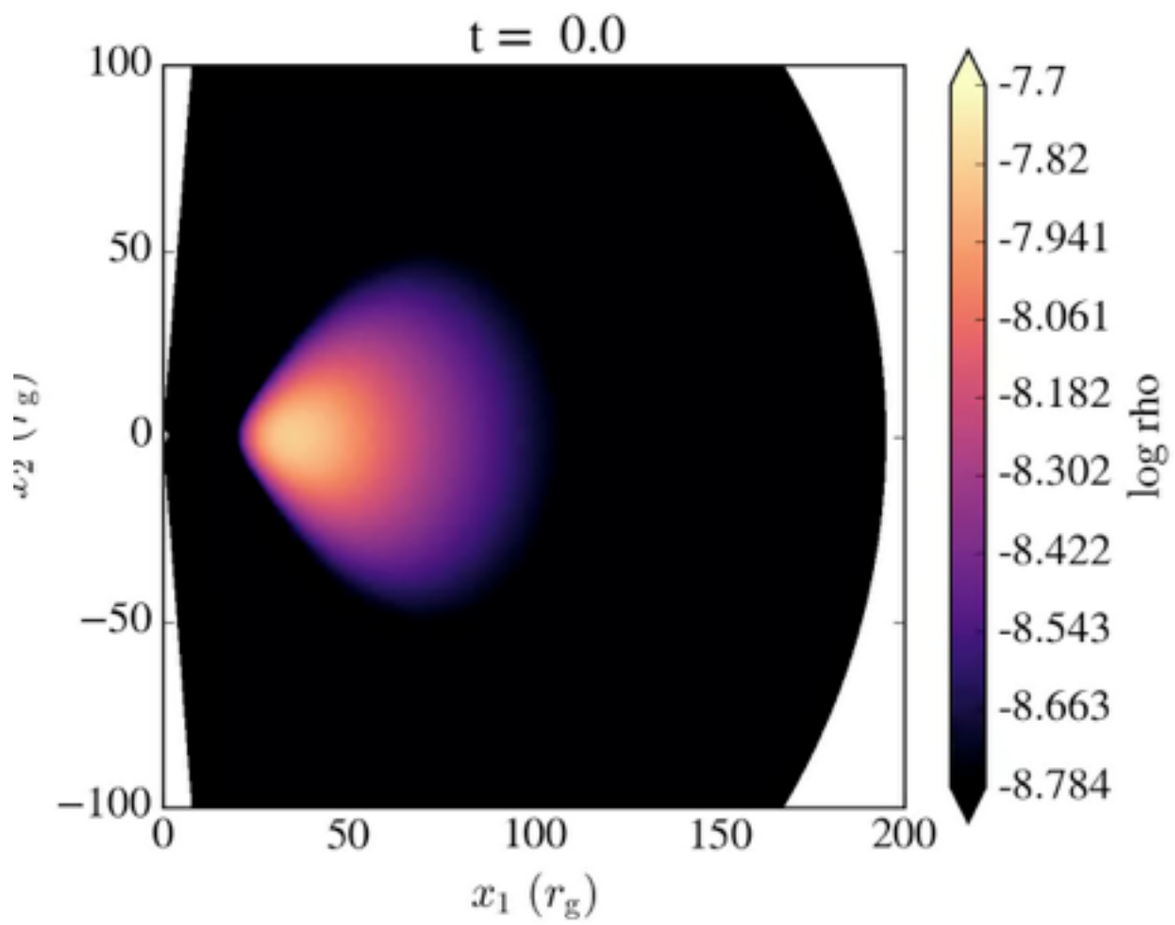
- Here is the relative error comparing several routines. The density is in cgs, the temperature in K.

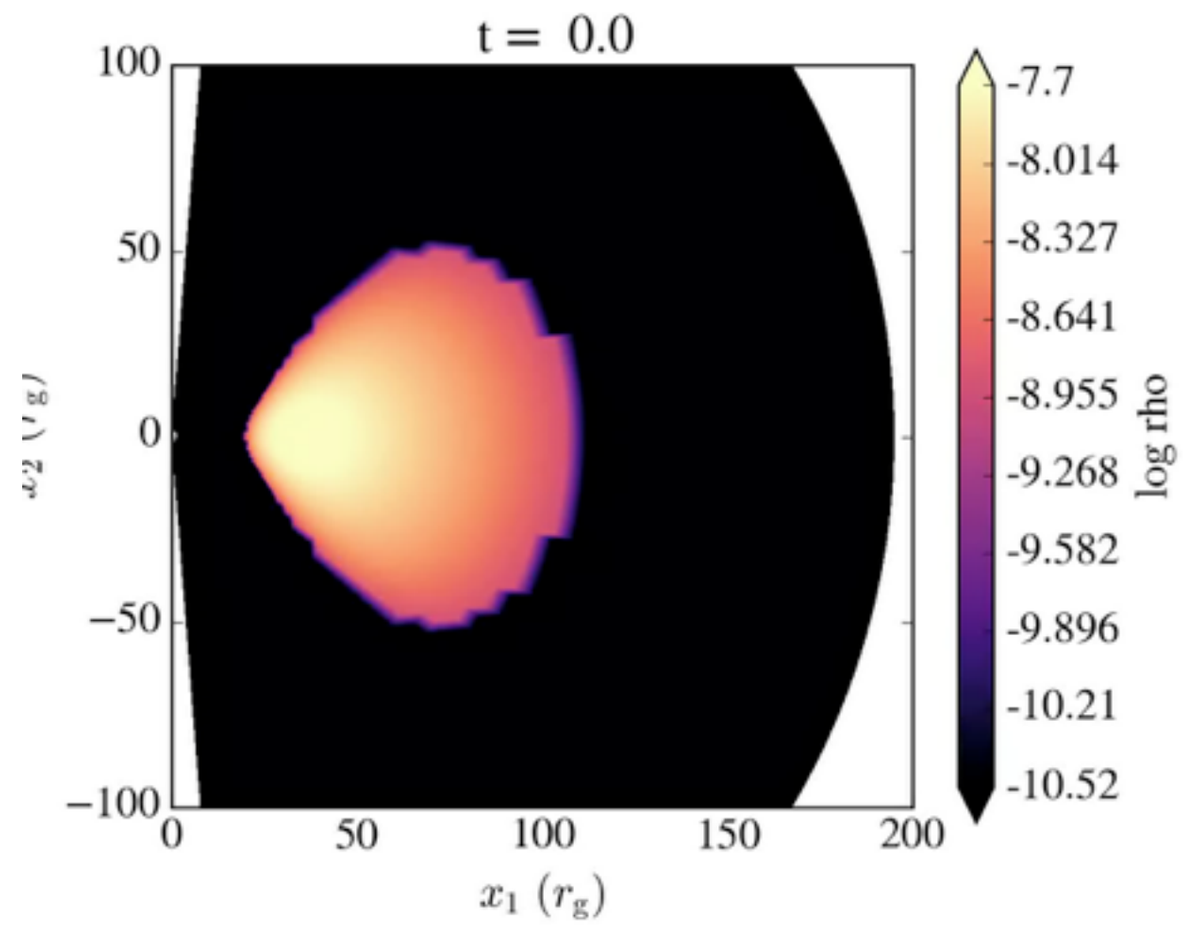


Based on Siegel et al. (2018)
Driver from O'Connor & Ott (2010),
Schneider et al. (2017)

Subtleties with the EOS: Atmosphere

Isentropic Fishbone Moncrief disk





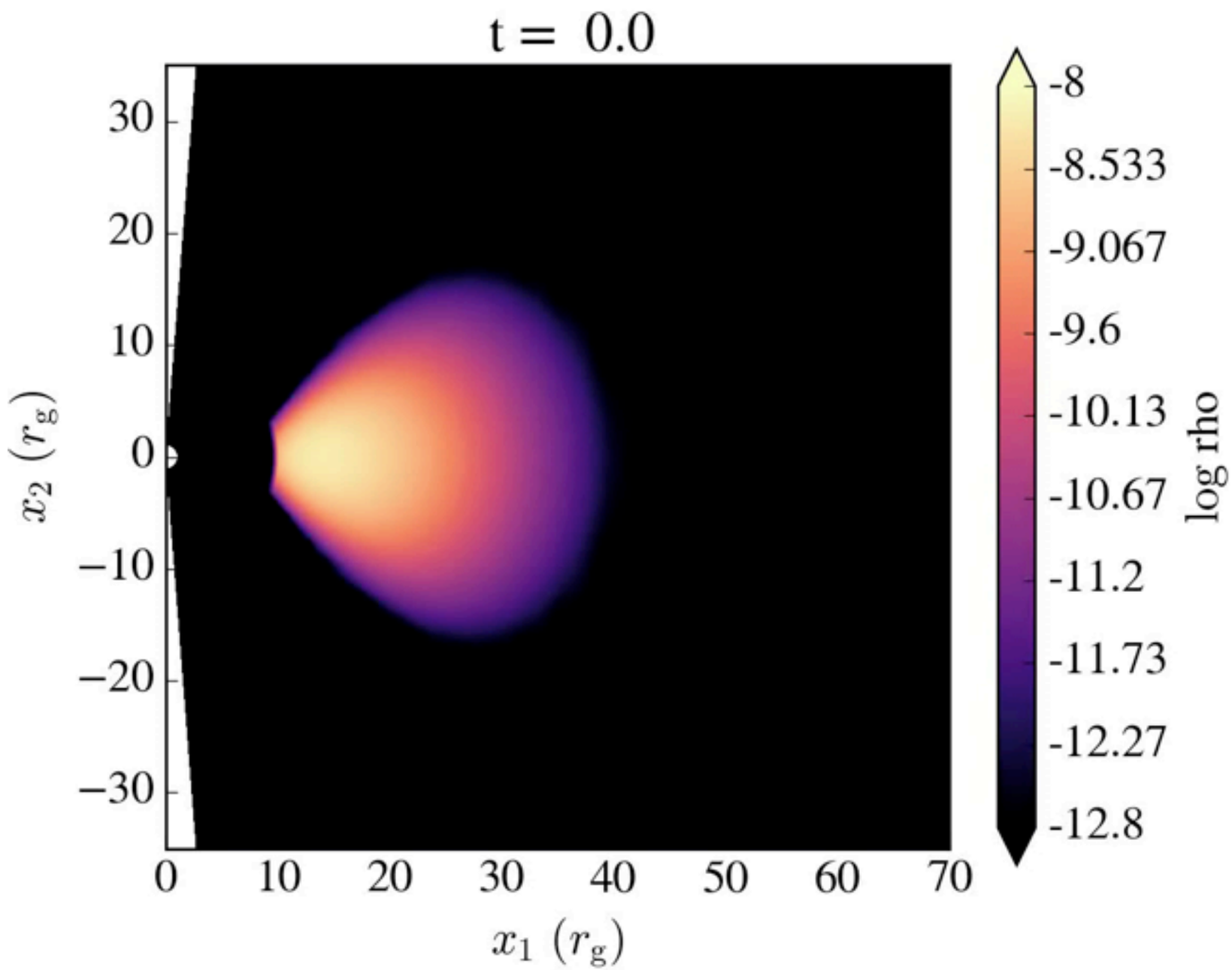
Atmosphere with same entropy as the disk and h=1

Atmosphere with higher entropy as the disk and h=1

In code units

Subtleties with the EOS: Atmosphere

Isentropic Fishbone Moncrief disk with outside enthalpy similar to minimum enthalpy in the disk



Leakage scheme

Charged beta-process

$$e^- + p \rightarrow n + \nu_e$$

$$e^+ + n \rightarrow p + \bar{\nu}_e$$

Plasmon decay

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Electron-positron pair annihilation

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$$e^- + e^+ \rightarrow \nu_x + \bar{\nu}_x$$

Absorption (opacity source)

$$\nu_e + n \rightarrow p + e^-$$

$$\bar{\nu}_e + p \rightarrow n + e^+$$

Scattering with free nucleons

Based on Ruffert et al. (1996)
Galeazzi et al. (2013)
Bruenn (1985) and other papers

Leakage scheme

Source terms

$$\nabla_\mu T^{\mu\nu} = Q u^\nu$$

Heating/cooling rate

$$\nabla_\mu (n_\mathrm{e} u^\mu) = R$$

Absorption/emission rate

$$R_\nu^\mathrm{eff} = \frac{R_\nu}{1 + \frac{t_\mathrm{diff}}{t_\mathrm{emission,R}}}$$

$$Q_\nu^\mathrm{eff} = \frac{Q_\nu}{1 + \frac{t_\mathrm{diff}}{t_\mathrm{emission,Q}}}$$

Based on Ruffert et al. (1996)
Galeazzi et al. (2013), with modifications from Rosswog & Liebendörfer (2003),Siegel & Metzger (2018), O'Connor & Ott (2010)

The effective rates are an interpolation between the optically thin and thick regime

$$t_\mathrm{diff} = \frac{6\tau^2}{c\kappa_{\nu_i}}$$

$$t_\mathrm{emission,R} = R_{\nu_i}/n_{\nu_i}$$

$$t_\mathrm{emission,Q} = Q_{\nu_i}/\varepsilon_{\nu_i}$$

Leakage scheme

$$R_\nu^{\rm eff} = \frac{R_\nu}{1 + \frac{t_{\rm diff}}{t_{\rm emission,R}}}$$

$$Q_\nu^{\rm eff} = \frac{Q_\nu}{1 + \frac{t_{\rm diff}}{t_{\rm emission,Q}}}$$

Based on Ruffert et al. (1996)
Galeazzi et al. (2013)

If the diffusion timescale is large (opaque region):

$$R_\nu^{\rm eff} = n_\nu / t_{\rm diff}$$
$$Q_\nu^{\rm eff} = \epsilon_\nu / t_{\rm diff}$$

$$\kappa(\bar{\nu}_e) = \kappa_s(\bar{\nu}_e, n) + \kappa_s(\bar{\nu}_e, p) + \kappa_a(\bar{\nu}_e, p)$$
$$\kappa(\nu_e) = \kappa_s(\nu_e, n) + \kappa_s(\nu_e, p) + \kappa_a(\nu_e, n)$$

In transparent region:

$$R_\nu^{\rm eff} = R_\nu$$

$$R_\nu = R_{\beta-\rm charged} + R_{\rm plasmon\ decay} + R_{e^-e^+}$$

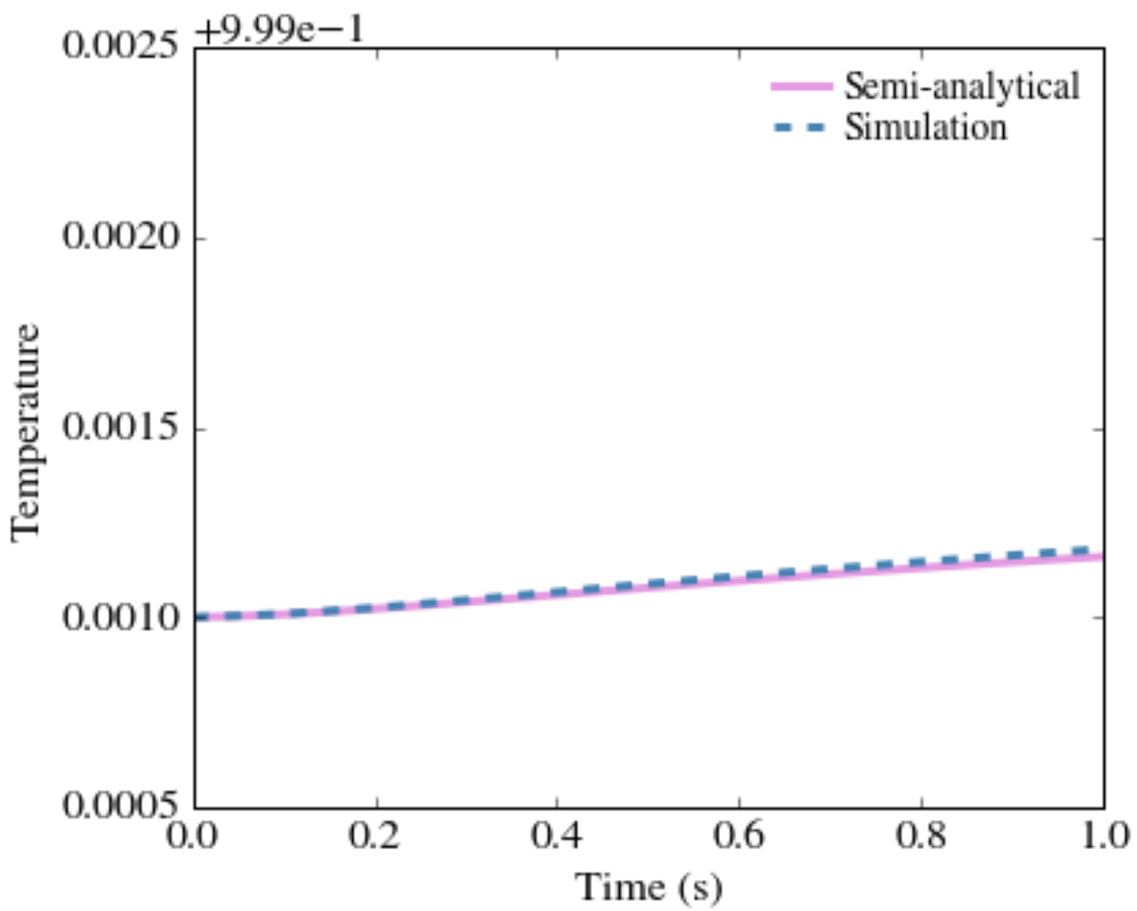
(same for Q)

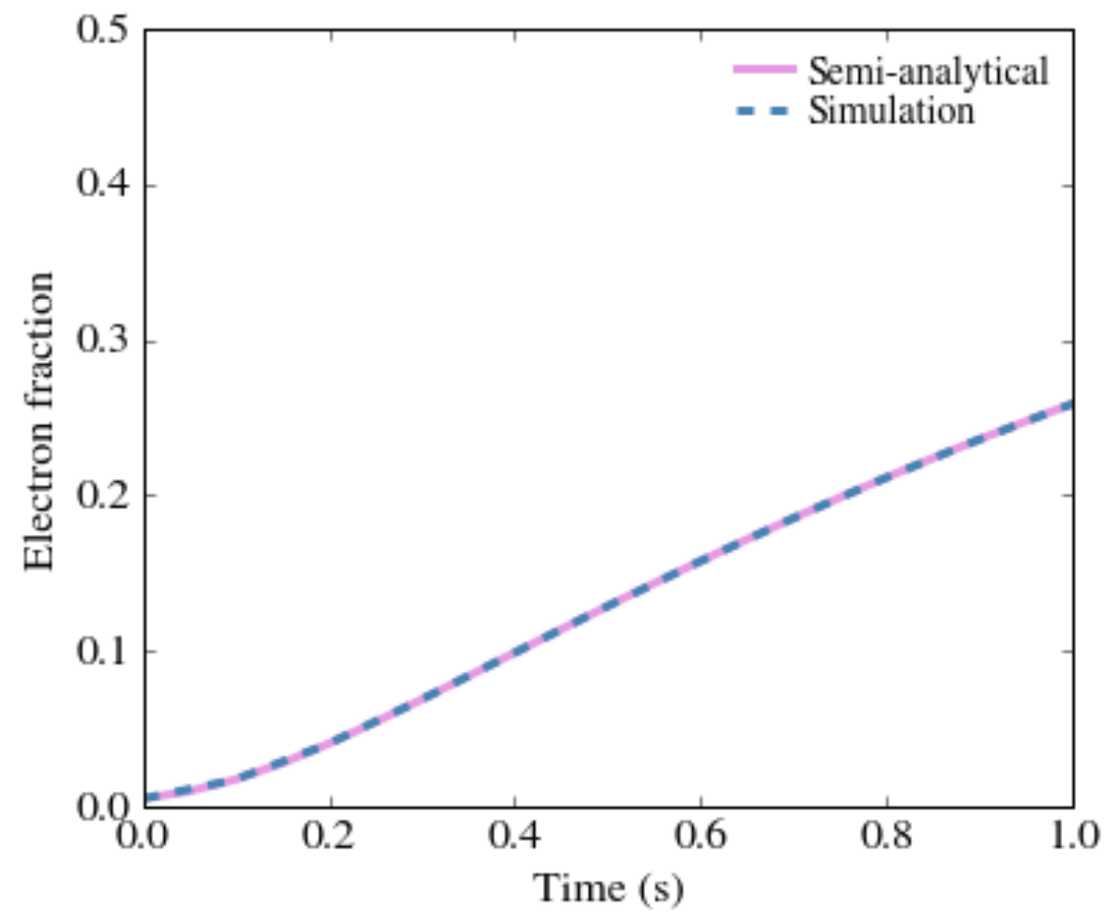
Leakage scheme testing

Evolution of isotropic, optically thin, constant density gas

$$\partial_t u = Q$$

$$\partial_t Y_e = R/\rho$$





Beta process for electron antineutrino

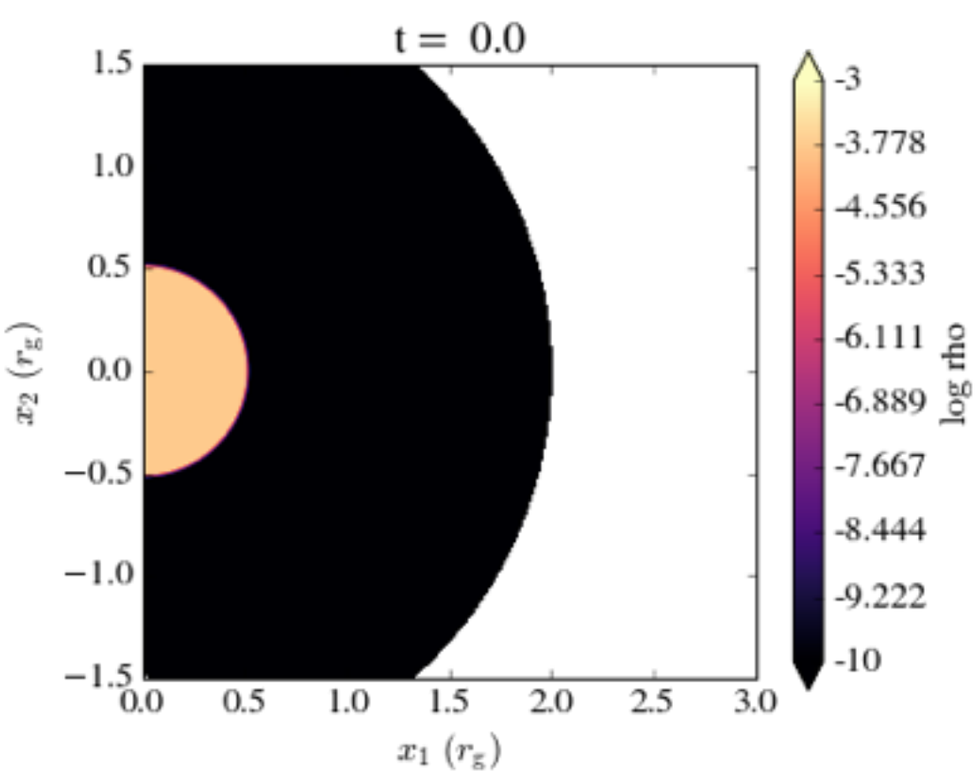
Ryan et al. (2015)
Miller at al. (2019)

Optical depth

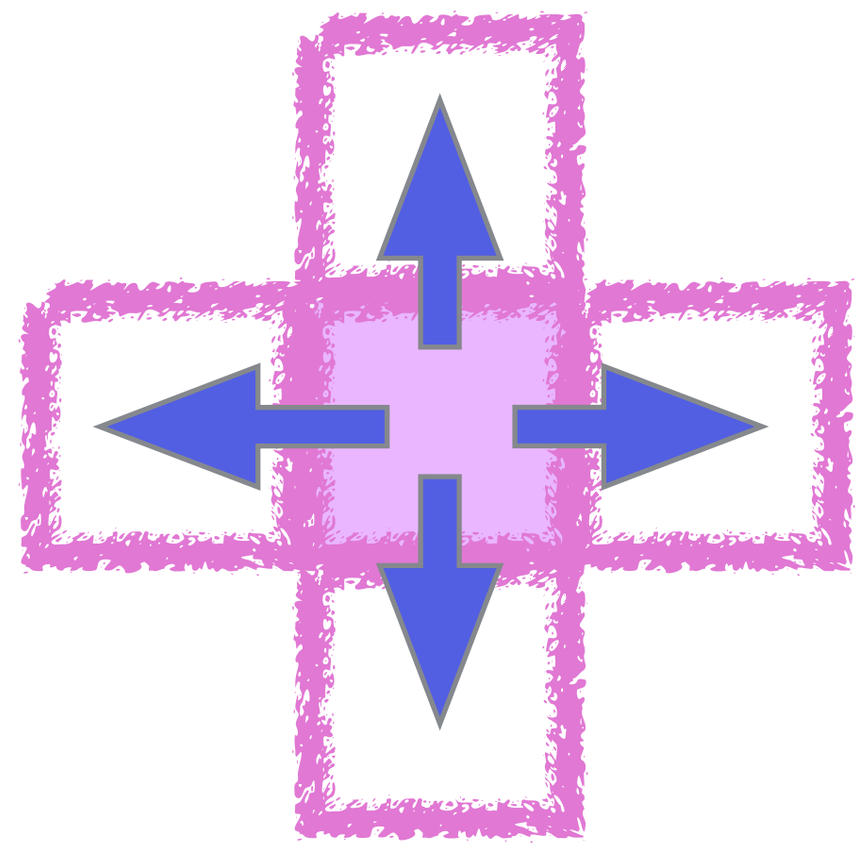
$$\tau = \int_{s_1}^{s_2} \kappa ds$$

Neilsen et al. (2014),
Siegel & Metzger (2018)

$$\min(\tau_{\nu,\mathrm{neigh}} + \bar{\kappa_\nu}(\bar{\gamma_{ab}} dx^a dx^b)^{1/2})$$



Comes into the calculation of the diffusion time



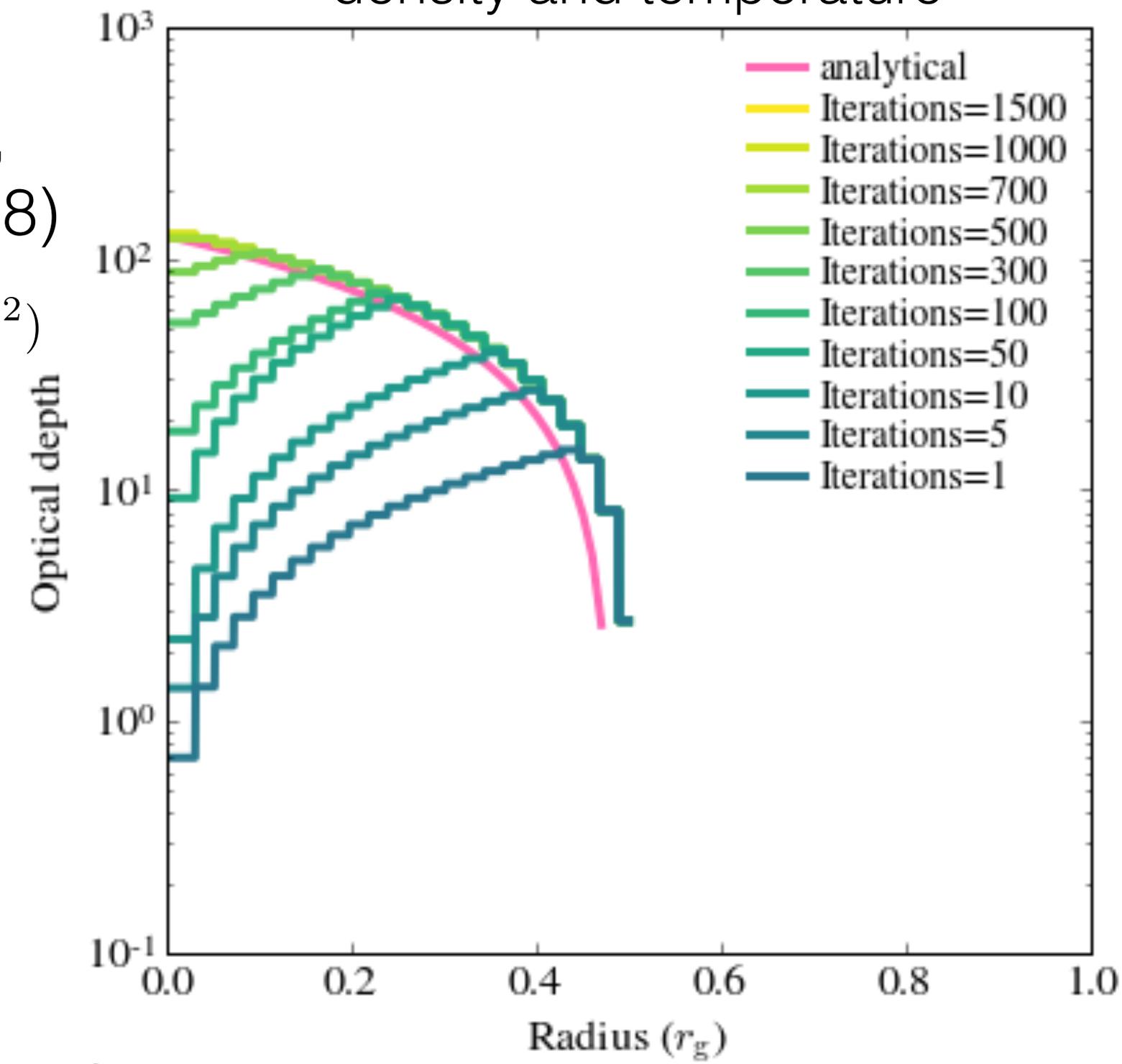
Leakage scheme: optical depth

$$\tau = \int_{s_1}^{s_2} \kappa ds$$

Neilsen et al. (2014),
Siegel & Metzger (2018)

$$\min(\tau_{\nu,\mathrm{neigh}} + \bar{\kappa_\nu}(\bar{\gamma_{ab}} dx^a dx^b)^{1/2})$$

Testing a sphere of constant density and temperature



Optical depth to electron neutrinos

Leakage scheme: optical depth

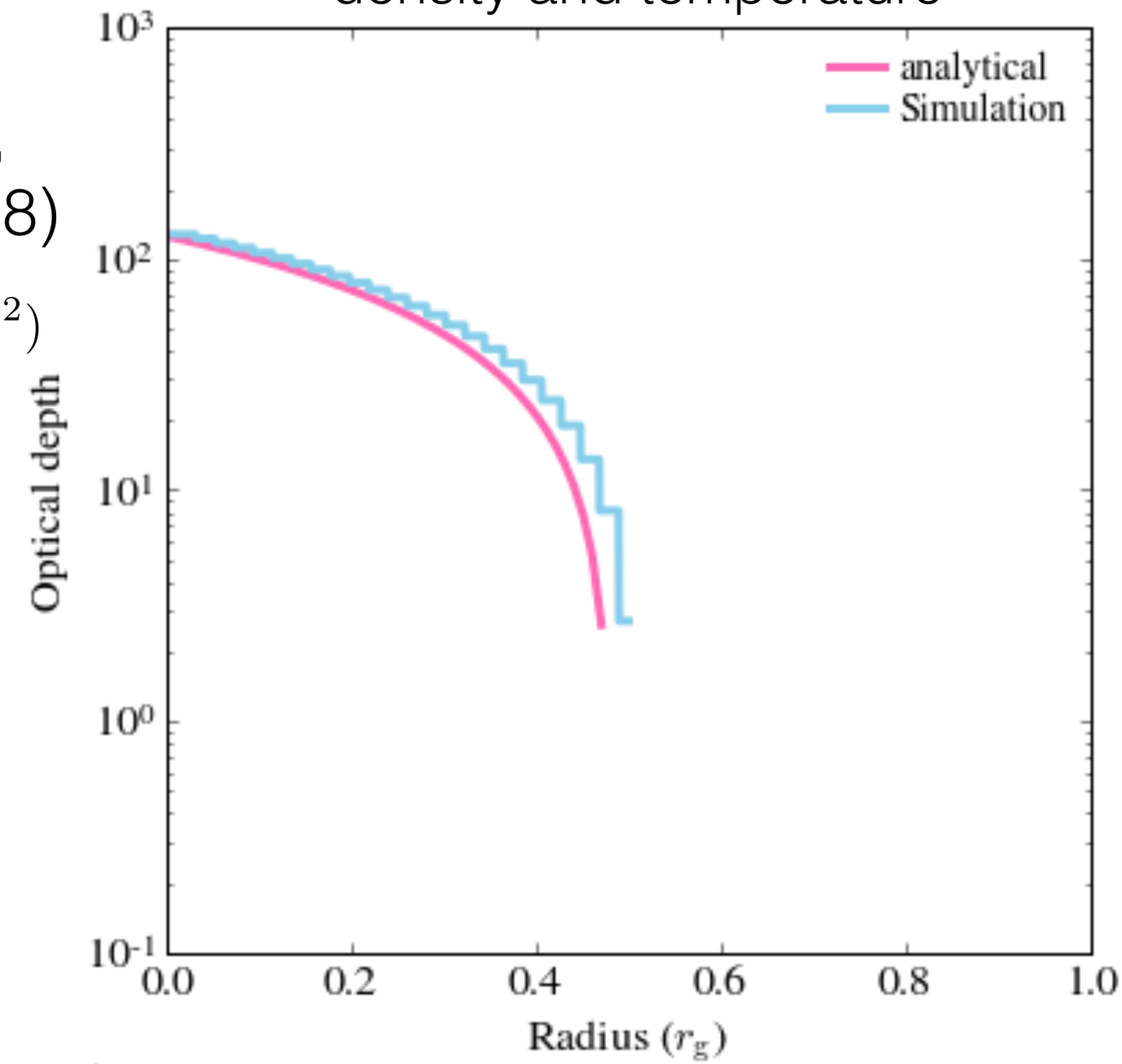
$$\tau = \int_{s_1}^{s_2} \kappa ds$$

Neilsen et al. (2014),
Siegel & Metzger (2018)

$$\min(\tau_{\nu,\mathrm{neigh}} + \bar{\kappa_\nu}(\bar{\gamma_{ab}} dx^a dx^b)^{1/2})$$

With our convergence criterion:

Testing a sphere of constant density and temperature



Optical depth to electron neutrinos

Conclusions and future work

- Tabulated EOS and neutrino leakage scheme are ready to work!
- Future work: We need to understand more the initial conditions in the atmosphere.
- Future work: add more robust utoprim routines
- Future work: work on using skynet to do the nucleosynthesis.

¡Gracias!