Numerical relativity in spherical coordinates in the Einstein Toolkit: SphericalNR

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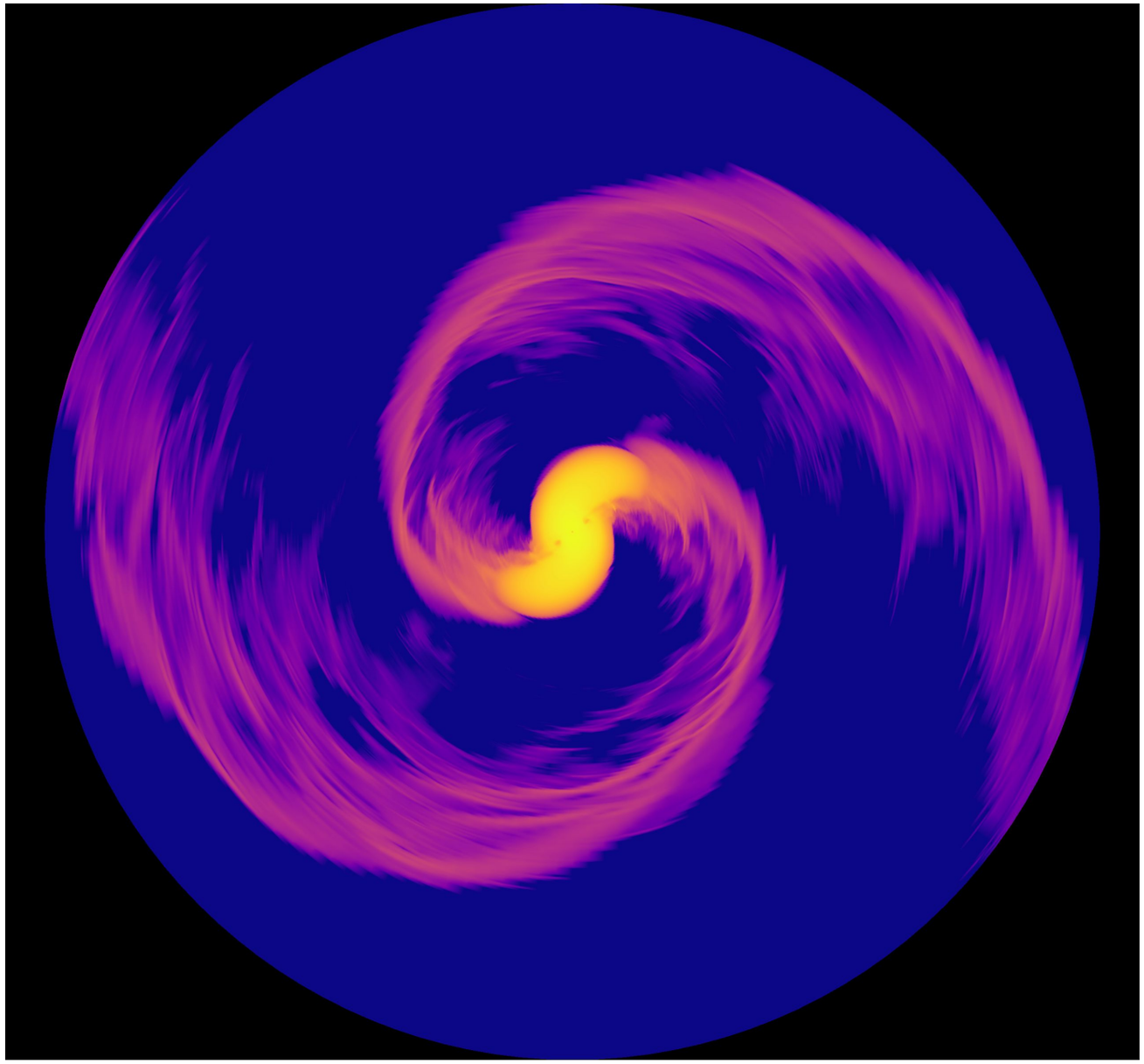

TCAN meeting
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Developing SphericalNR, a new framework for numerical relativity in spherical coordinates with the Einstein Toolkit

Mathematical formulation:

Need to **solve hyperbolic PDEs** using **finite differences** and **finite volume methods** in **spherical coordinates** without symmetry assumptions and regularization at the origin and axis.

Algorithmic development:

Einstein Toolkit is inherently Cartesian, develop **boundary condition algorithm** to enable the use of spherical coordinates while being able to **use as much existing infrastructure as possible**.

Filtering to ameliorate **severe CFL restrictions** for high resolution 3D evolutions in spherical coordinates.

Automatic code generation for RHS: Use python to generate the extremely complex RHS of the Einstein field equations and GRMHD equations.

SphericalNR: Numerical relativity in spherical coordinates with the Einstein Toolkit: **A quick overview**

Covariant form of **BSSN equations** [Brown 2009] (see also [Alcubierre et al 2011]) using a **spherical coordinate reference metric** where the conformally related metric is expressed as the sum of a **fixed background metric plus corrections** [Montero et al 2012, Baumgarte et al 2013, Baumgarte et al 2015]. Uses **NRPy+** [Ruchlin et al 2018].

[**Mewes et al 2018**: https://arxiv.org/abs/1802.09625]

fCCZ4 (constraint damping) and **GRMHD** (built on GRHydro, cell-centered vector potential evolution) in spherical coordinates.

[**Mewes et al 2020**: https://arxiv.org/abs/2002.06225]

Filtering to ameliorate **severe CFL restrictions** for high resolution 3D evolutions in spherical coordinates using **FFT filters in both theta and phi**. Support for **arbitrary radial coordinates** (generic Fisheye coordinates, exp(x1)) that can be used in evolution.

[**Mewes et al 2021**: in prep.]

SphericalGRHydro: A cell-centered vector potential GRMHD code in spherical coordinates

Implemented GRMHD reference metric formalism in **GRHydro**, an open source GRMHD code in the Einstein Toolkit:

- **Cell-centered Vector potential evolution** in generalized Lorenz gauge (see IGM and Spritz) with higher order fd
- all variables are **properly rescaled** (evolve vector components in orthonormal basis w.r.t. spherical reference metric)
- replace partial derivatives with covariant derivatives of the background metric
- **minimal changes to GRHydro**, only in **source terms (NRPy+ generated)**, pass rescaled ADMBase metric to GRHydro
- Code is logically Cartesian, so **Reconstruction**, **Riemann solvers** and similar **infrastructure already present** in GRHydro can be used **out-of-the-box**

Double FFT filter to damp CFL-unstable modes and alleviate prohibitively small timesteps in full 3D simulation

Cell volumes are **not constant** in spherical coordinates, and decrease as the origin and axis are approached, **leading to prohibitive timestep restrictions** when using high angular resolution grids.

Perform **adaptive FFT filtering** by **damping CFL unstable modes** at given radius and latitude.

Filter in phi is trivial as coordinate is periodic, to filter in theta extend great circles from [0,pi] to [0,2pi] by using the data on the continued great circle multiplied by the appropriate axis parity factors.

$$\mathbf{X}(x_1, \vartheta, \phi) = \begin{cases} \mathbf{Y}(x_1, \theta, \phi), & \vartheta \in [0,\pi] \\ (-1)^a \mathbf{Y}(x_1, \pi - \theta, \pi + \phi), & \vartheta \in [\pi,2\pi] \end{cases}$$

$$\tilde{\mathbf{X}}(x_1, l, \varphi) \rightarrow \tilde{\mathbf{X}}(x_1, l, \varphi) \begin{cases} 1, & |l| \leq l_{\max} \\ e^{-(|l| - l_{\max})}, & l > l_{\max} \end{cases}, \quad l_{\max} = \max(\frac{2r}{dr} n_\theta, 2)$$

$$\tilde{\mathbf{X}}(x_1, \theta, m) \rightarrow \tilde{\mathbf{X}}(x_1, \theta, m) \begin{cases} 1, & |m| \leq m_{\max} \\ e^{-(|m| - m_{\max})}, & m > m_{\max} \end{cases}, \quad m_{\max} = \max(\frac{2r}{dr} n_\theta \sin\theta, 2)$$

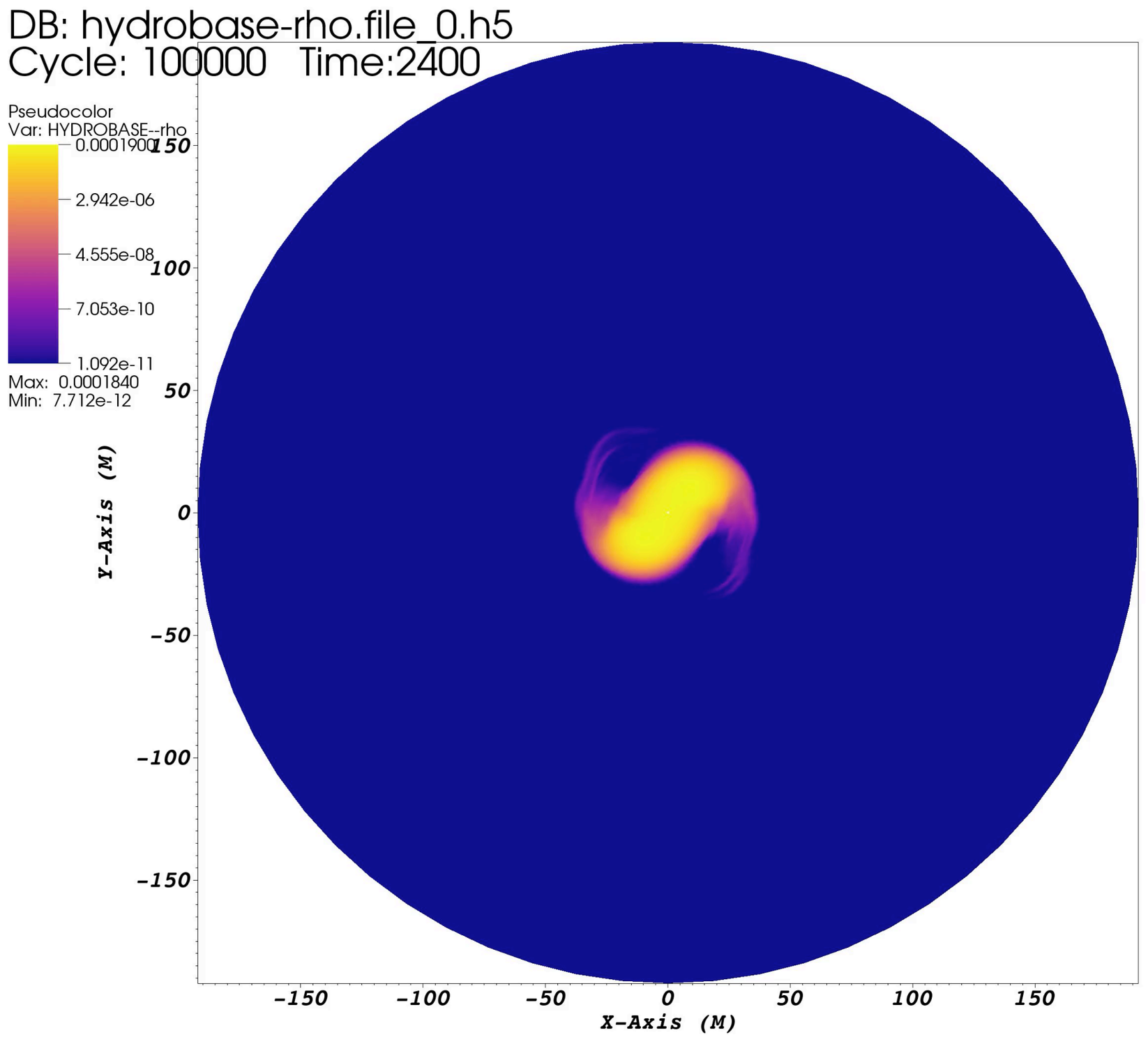
Automatic code generation with NRPy+

```
Thetarhs = Rational(1,2) * alpha * (psim4*(traceRbar
         - 8*DbarphiDbarphicontraction - 8*Dbar2phicontraction)
         + Rational(2,3) * trK*trK - 2*Theta*trK)

for i in range(3):
    for j in range(3):
        Thetarhs -= Rational(1,2) * alpha * AbarDD[i][j] * AbarUU[i][j]

for i in range(3):
    Thetarhs -= ZU[i]*alphadD[i]

Thetarhs -= kappa1*(2+kappa2)*Theta + couple_matter * 8*M_PI*alpha*E

Thetarhs += LbetaTheta

Thetarhs *= fCCZ4
```

```
gfs_rhs[THETA][idx] = fCCZ4*(ThetadD0*tmp116 + ThetadD1*tmp82 + ThetadD2*tmp112 -
alphadD0*tmp276 - alphadD1*tmp301 - alphadD2*tmp319 - tmp457*tmp651 - tmp459*tmp65
1 - tmp460*tmp650 - tmp461*tmp650 - tmp462*tmp652 - tmp464*tmp652 - tmp465*tmp653
- tmp467*tmp653 - tmp468*tmp650 - tmp493*tmp617 - tmp568*(kappa2 + 2) + tmp650*((2
.0/3.0)*tmp454 + tmp569 + tmp596));
```

Write equations in **tensorial notation** in **python scripts**, **NRPy+ generates c code** include files at arbitrary finite-difference order to be used in for loops.

Results: Magnetized model U11, bar mode instability, filtering theta and phi



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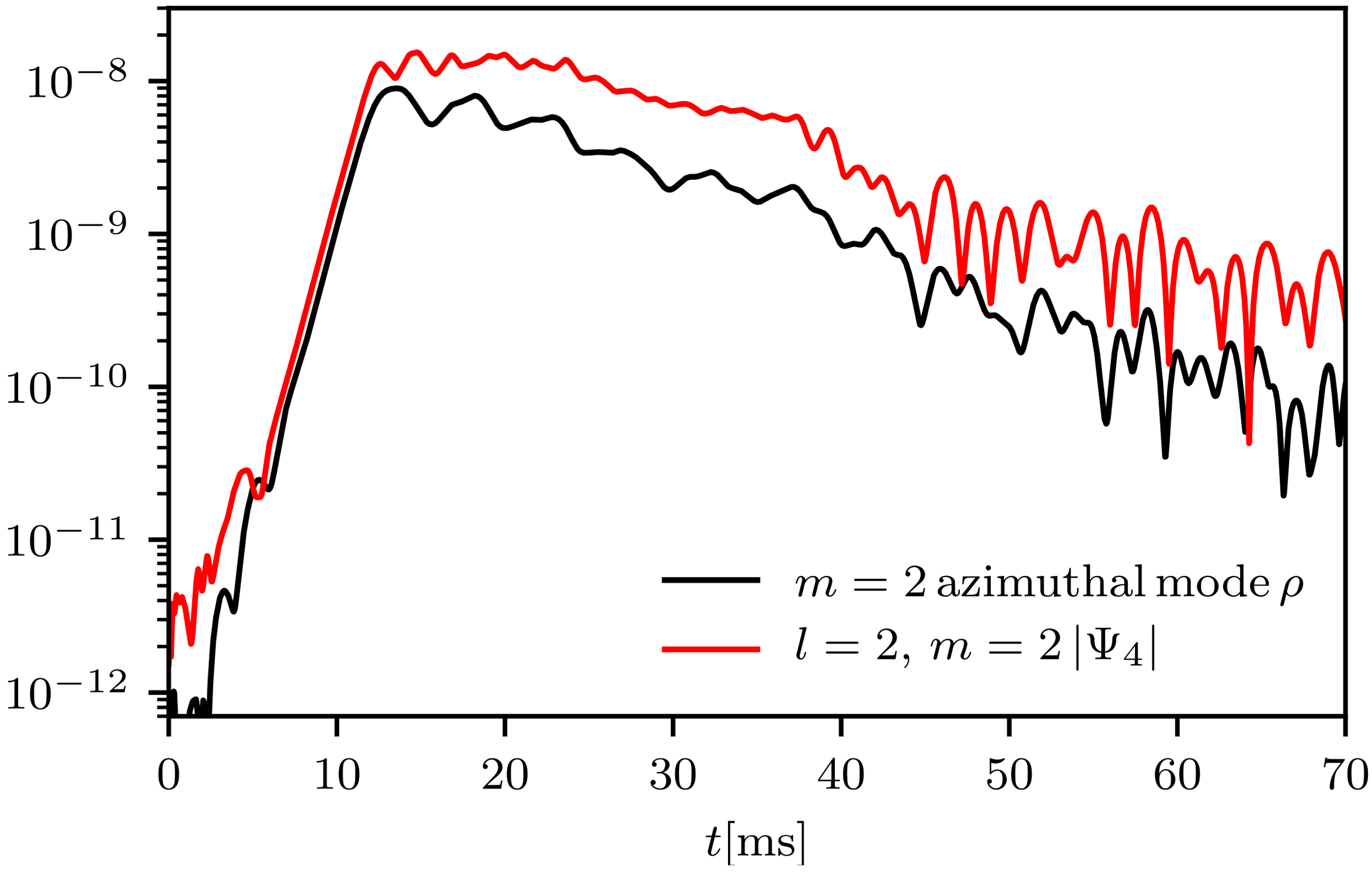
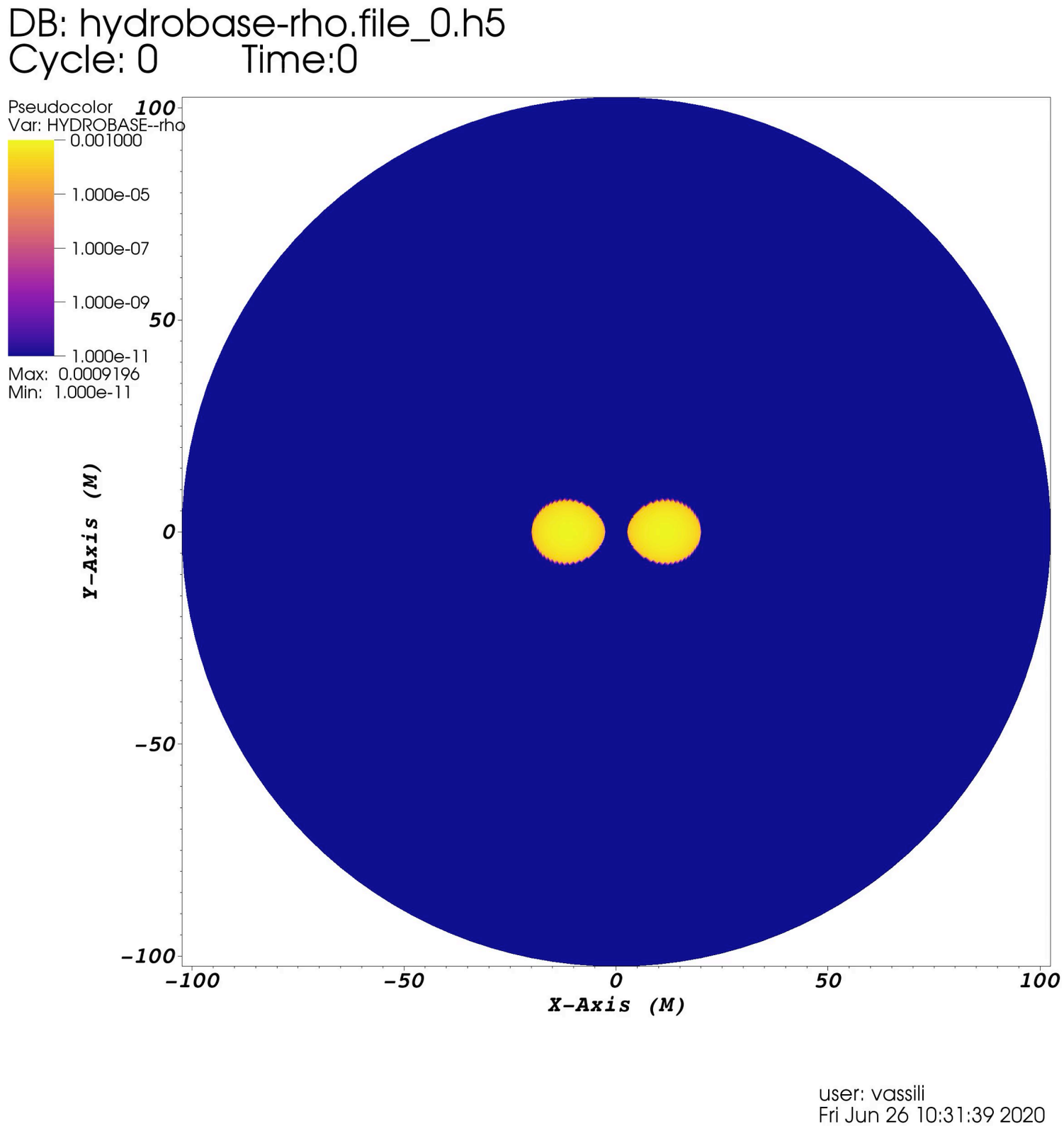
Results: Magnetized model U11, connecting the bar mode instability to gravitational wave emission



Results: 4 M_sun BNS merger, Gamma=2.5, co-rotating BNS, filtering in theta and phi

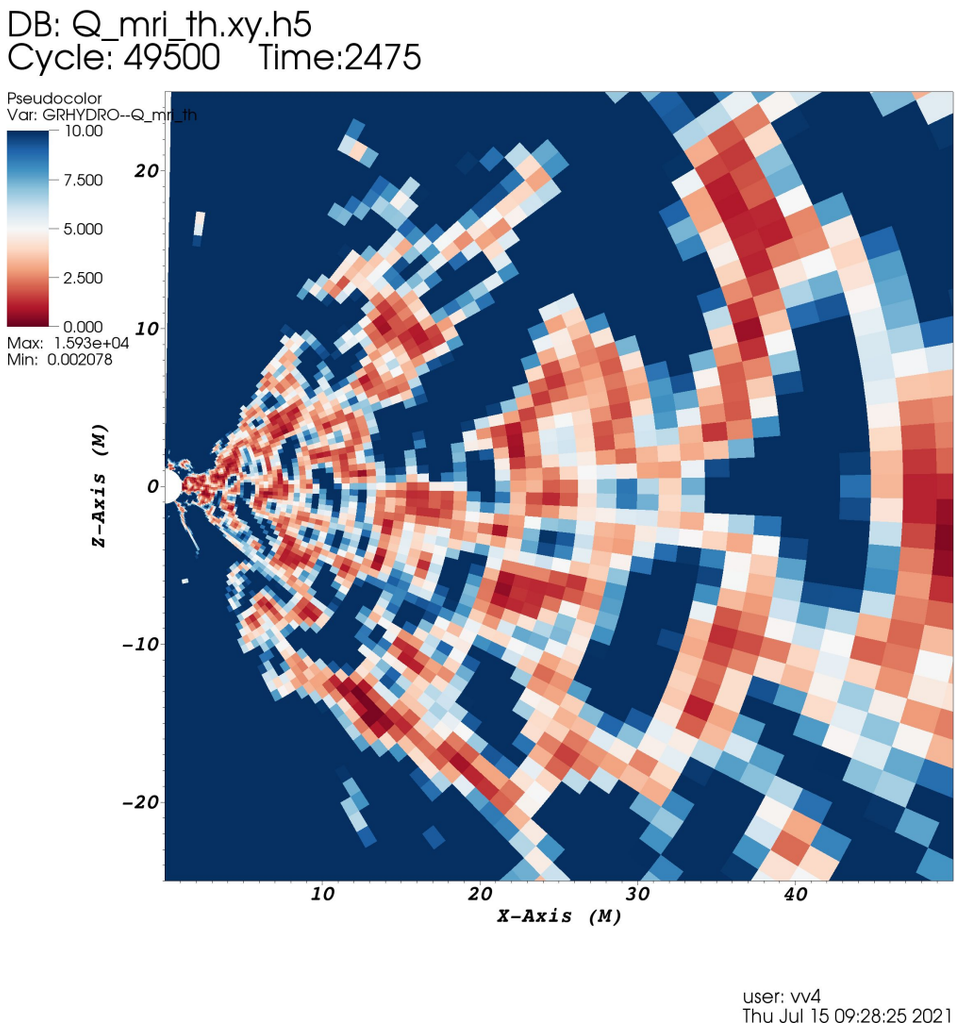


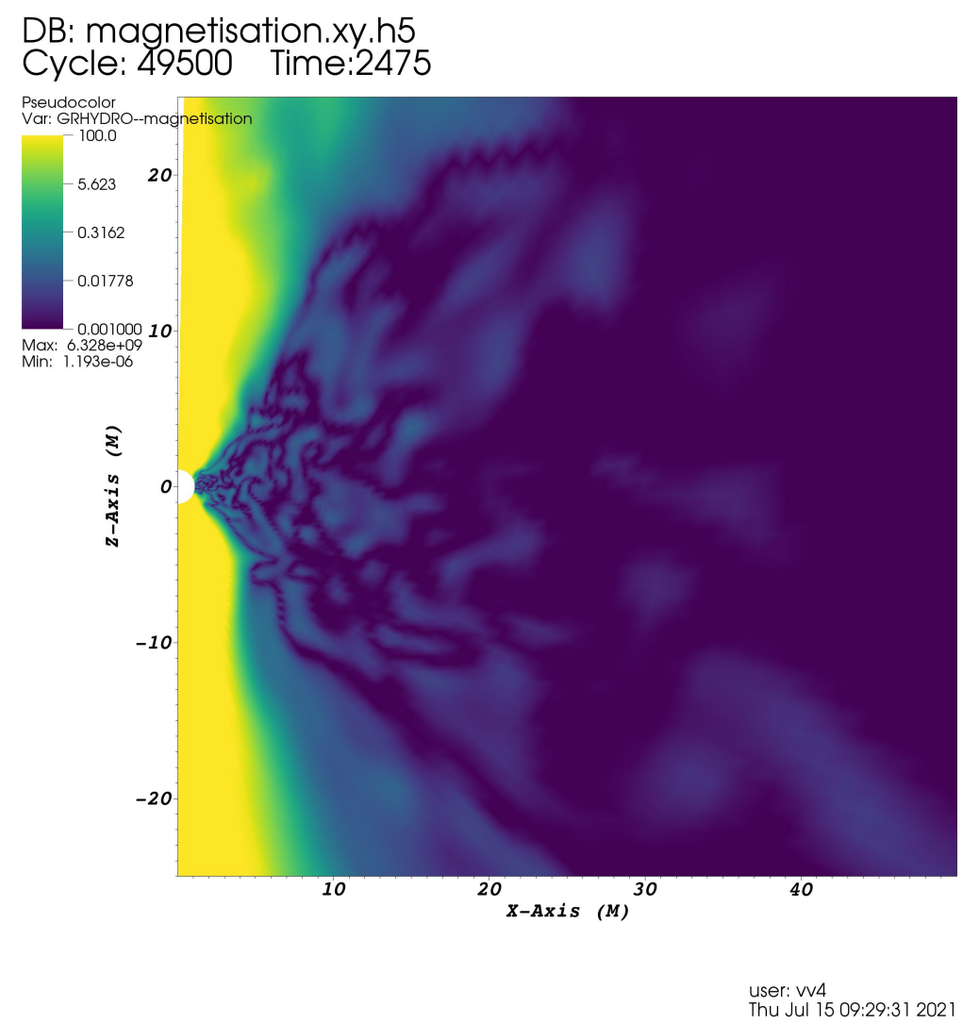
New developments for SphericalNR: trying to run a Fishbone-Moncrief torus

9th order WENO-Z reconstruction + **10th order fd** in curl of vector potential and evolution equation for vector and EM scalar potentials.

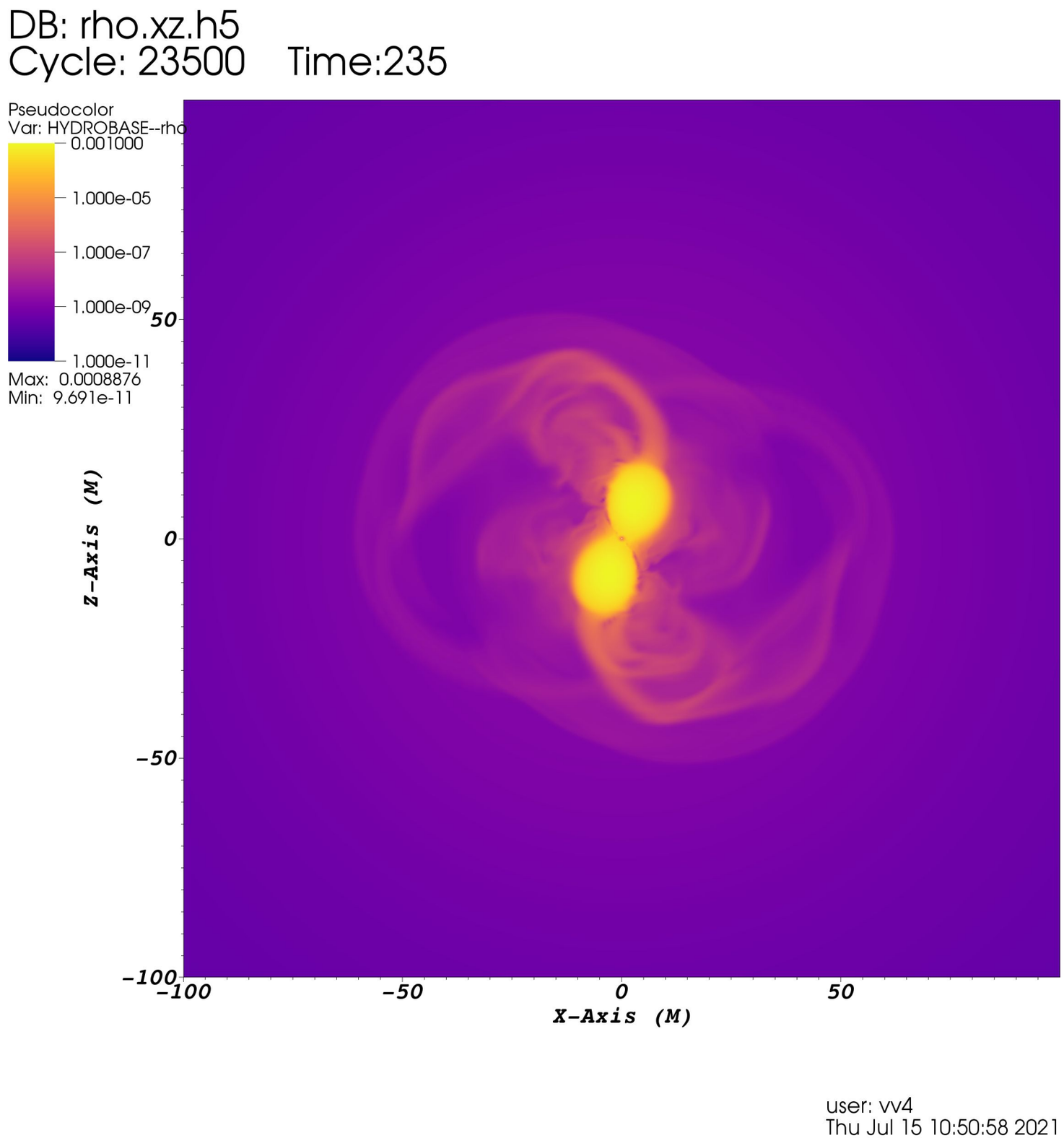
Original **atmosphere treatment** in GRHydro **completely revamped**, now being able to use **radially dependent floors**.

Evolving **entropy evolution equation** and use it as a fallback in **primitive recovery**.





Same 4 M_sun BNS merger, Gamma=2.5, co-rotating BNS, filtering in theta and phi with new developments



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Thank you for your attention!