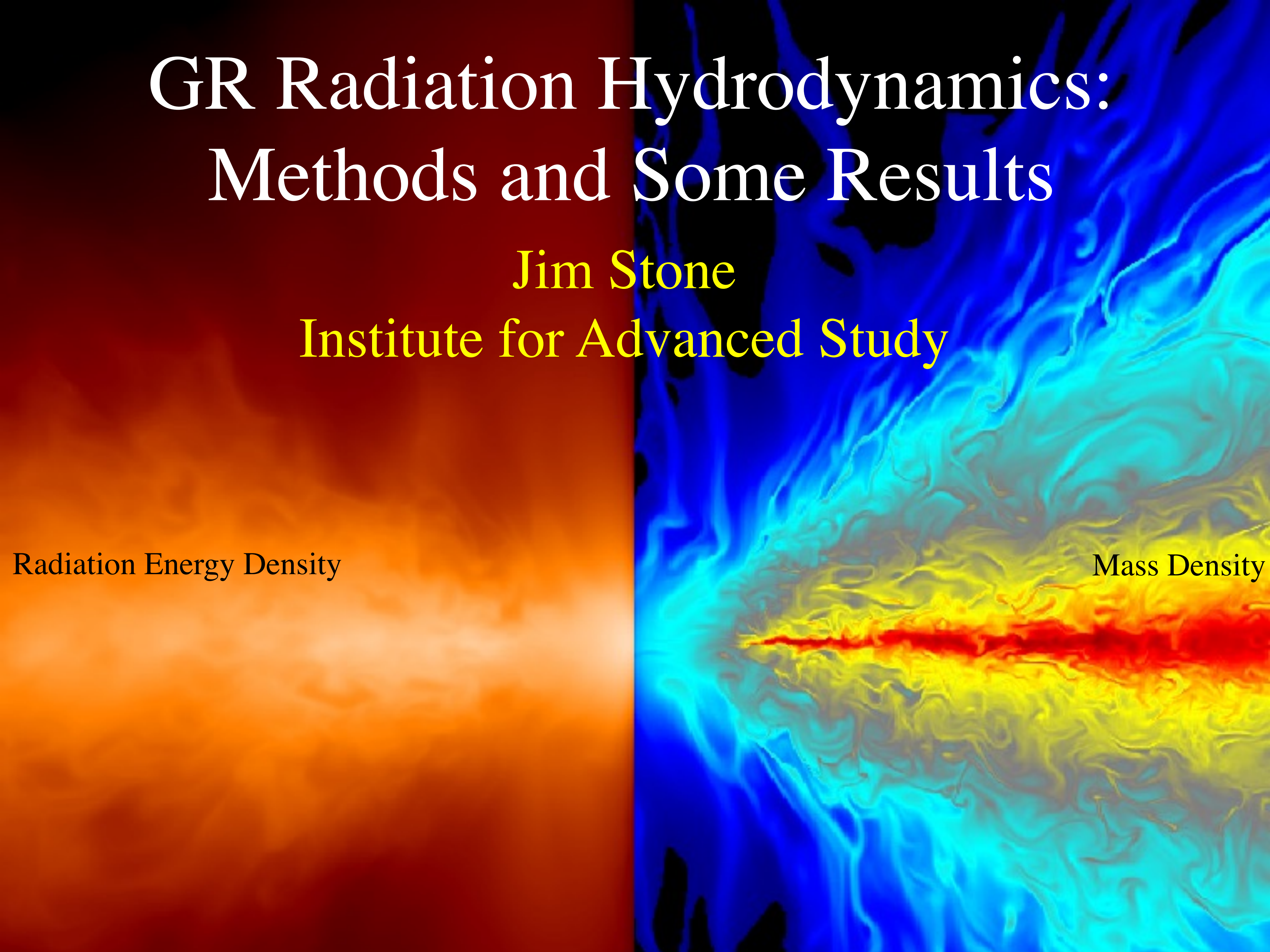
GR Radiation Hydrodynamics: Methods and Some Results

Jim Stone
Institute for Advanced Study



Motivation

In black hole accretion disks, radiation pressure exceeds gas pressure inside

$$r/R_G < 170(L/L_{\rm Edd})^{16/21}(M/M_\odot)^{2/21}$$

(Shakura & Sunyaev 1973). Radiation needs to be included in dynamical models when $L/L_{Edd} > 0.01$.

Even when $L/L_{Edd} < 0.01$, radiation can be important

- for energy transport, e.g. Compton cooling,
- when the plasma is only partially ionized, leading to enhanced opacities, e.g. line-driven winds.

For these reasons, it is widely appreciated radiation is important in accretion onto compact objects.

- **This talk:** selected overview of computational methods and issues for radiation hydrodynamics.

- Cannot review all progress in the field.

- Similar methods and issues apply for ***neutrino transport***, but will not discuss here.

Outline

1. Motivation
2. Numerical methods
3. Tests
4. Challenging issues
5. Summary

2. Numerical Methods

In general, must solve equations of relativistic hydrodynamics

$$(\rho u^{\mu})_{;\mu} = 0$$

$$(T^{\mu}{}_{\nu} + R^{\mu}{}_{\nu})_{;\mu} = 0$$

where the radiation stress-energy tensor is composed of moments of the specific intensity I_ν

$$R^{\mu}{}_{\nu} = \frac{1}{h^4} \int \frac{d^3p}{\sqrt{-g}p^t} p^{\mu} p_{\nu} \left(\frac{I_\nu}{\nu^3}\right)$$

Requires solving the equation of radiation transport for I_ν (Mihalas & Mihalas 1984)

$$\frac{D}{ds}\left(\frac{I_\nu}{\nu^3}\right) = \left(\frac{j^{\rm a}_\nu}{\nu^2}\right) + \left(\frac{j^{\rm s}_\nu}{\nu^2}\right) - \left(\nu\alpha^{\rm a}_\nu\right)\left(\frac{I_\nu}{\nu^3}\right) - \left(\nu\alpha^{\rm s}_\nu\right)\left(\frac{I_\nu}{\nu^3}\right)$$

Thermal emission and effective emission due to scattering

Absorption

Effective absorption due to scattering

Solving the invariant transfer equation is expensive, so simpler approximations are generally adopted.

In the fluid frame, radiation stress-tensor can be expressed in terms of energy density E, fluxes F, and pressure tensor P (Sadowski et al. 2013)

$$R = \left[\begin{array}{cc} E & F^i \\ F^j & P^{ij} \end{array} \right]$$

Convenient to express ignorance of fluid-frame radiation pressure tensor in terms of Eddington tensor $f = P / E$.

Then, approximate methods can be developed by approximating f. Simplest example, in very optically thick flows Eddington approximation applies: $f^{ij} = \delta^{ij}/3$

Approximate Methods: Flux-limited diffusion (non-relativistic flows)

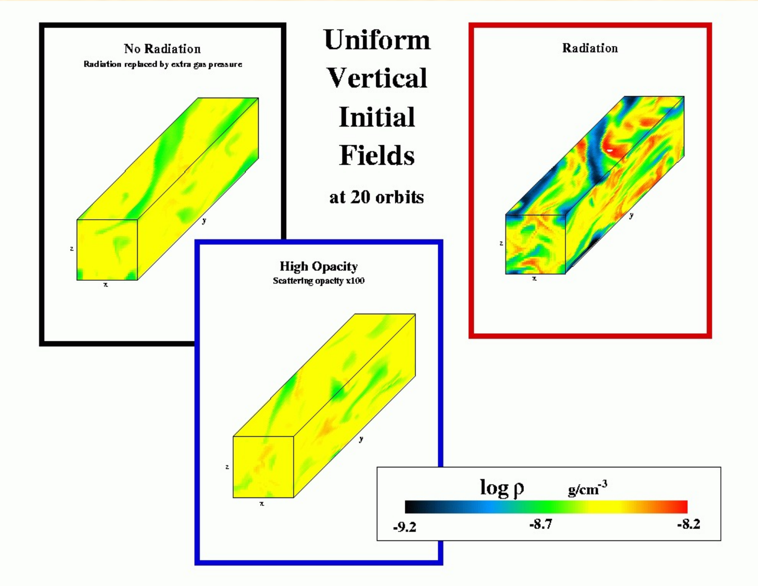
Assume radiation flux given by Ficks's Law

$$\mathbf{F} = -\frac{c\lambda}{\chi}\nabla E$$

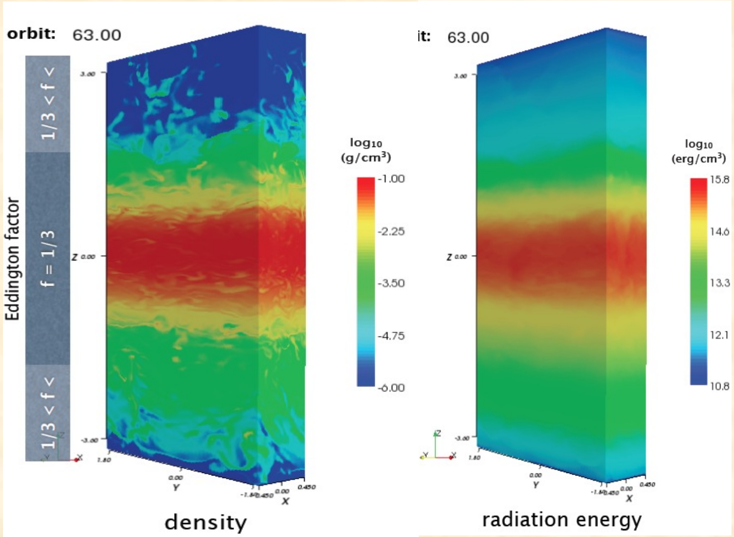
$\lambda=\lambda(E)$ is limiter that prevents super-luminal transport in optically thin regions

Reduces equations to two-temperature diffusion approximation.

FLD has been used to study many problems in non-relativistic flows, e.g. saturation of MRI in local shearing-box simulations.



Turner et al. (2002; 2003)



Hirose et al. (2009)

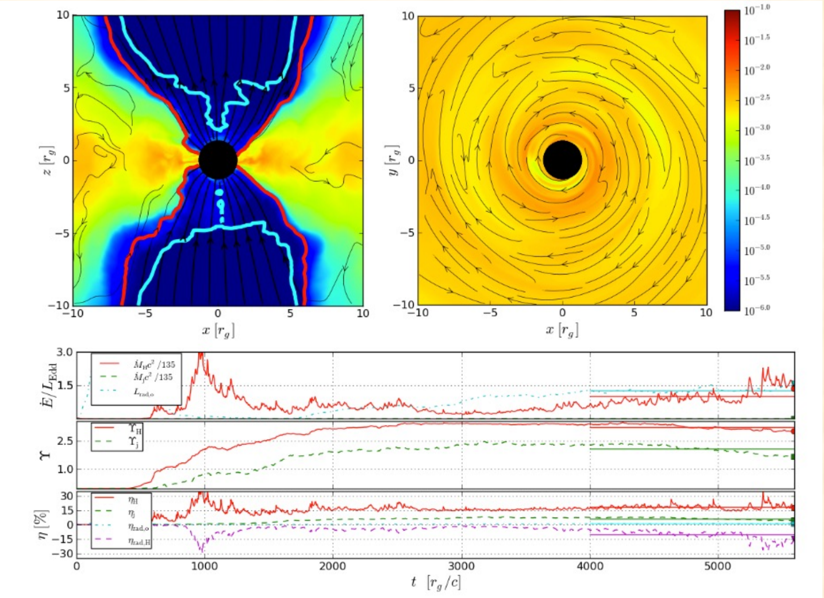
Approximate Methods: M1 Closure

Assume radiation pressure tensor is isotropic in a “rest frame” in which flux is zero. Leads to analytic form for Eddington tensor.

$$\mathsf{f} = \frac{1-\xi}{2}\mathsf{I} + \frac{3\xi-1}{2}\mathbf{n_F}\otimes\mathbf{n_F}, \qquad \xi = \frac{3+4\parallel \mathbf{F_r}^{\mathbf{n}}/(\mathbf{c}\mathbf{E}_\mathbf{r}^\mathbf{n}) \parallel^\mathbf{2}}{5+2\sqrt{4-3\parallel \mathbf{F_r}^{\mathbf{n}}/(\mathbf{c}\mathbf{E}_\mathbf{r}^\mathbf{n}) \parallel^\mathbf{2}}}$$

First global radiation GR-MHD simulations by McKinney et al. (2014; 2015) and Sadowski et al. (2014; 2015) using M1 closure

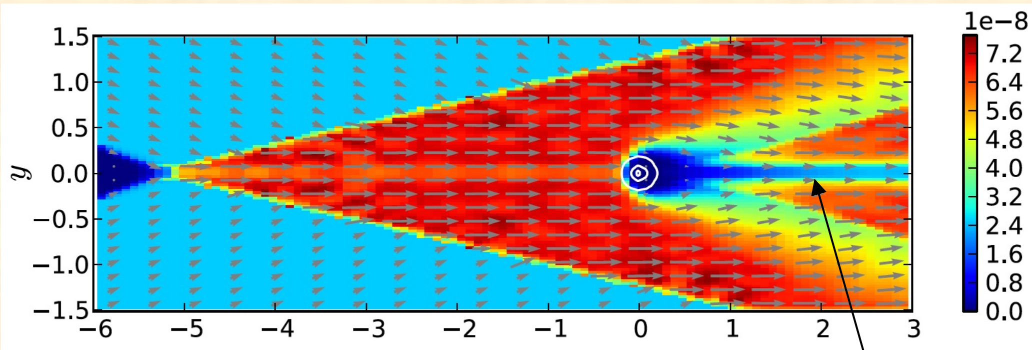
McKinney et al. (2014)



Now widely used by many groups for many applications…

But there are problems with M1, e.g. photon rays "collide" and merge.

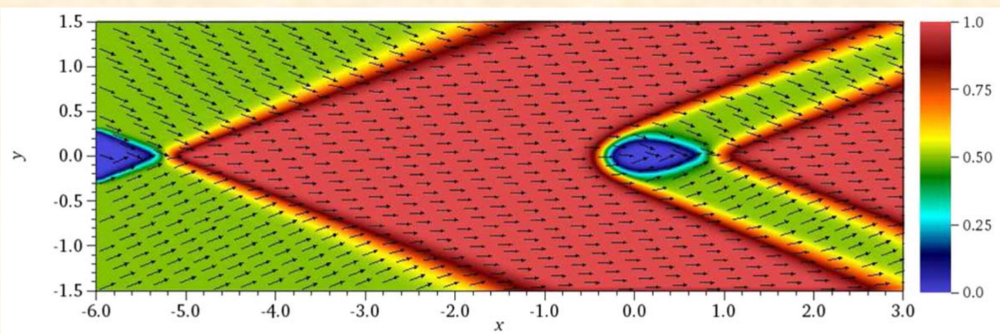
Shadow test using two beams:



Sadowski et al. (2013)
McKinney et al. (2014)

Note third shadow created by merged beams

Correct answer



Anninos & Fragile (2021)
M1 method using two different
(non-interacting) frequencies

Approximate Methods: variable Eddington tensor (VET) methods

Components of the Eddington tensor can be calculated directly from the specific intensity

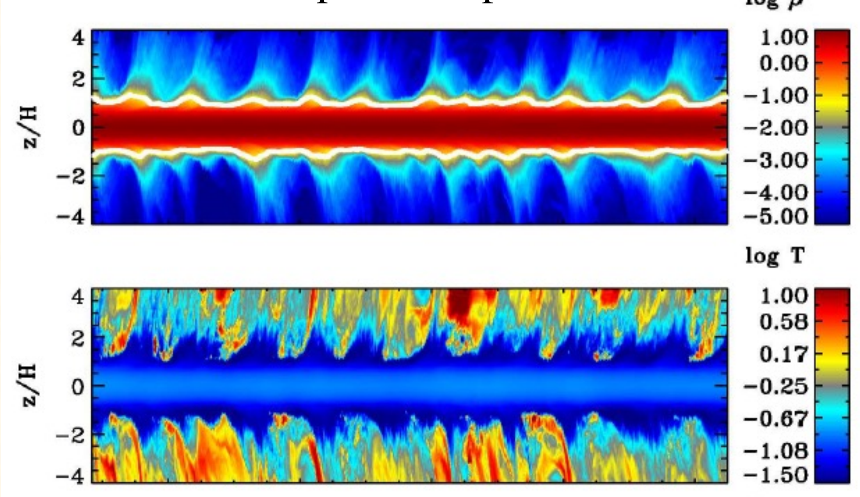
$$f^i{}_j = \frac{\int d\nu d\Omega I_\nu n^i n_j}{\int d\nu d\Omega I_\nu}$$

Use approximate methods to compute I_ν, for example:

- Time-independent transfer equation for non-relativistic flows (Davis et al 2012, Jiang et al 2012)
- Monte-Carlo methods, e.g. MOC-MC (Ryan & Dolence 2020)

VET methods have been used by Jiang et al. (2014) to study corona above gas-pressure dominated MRI-unstable disks.

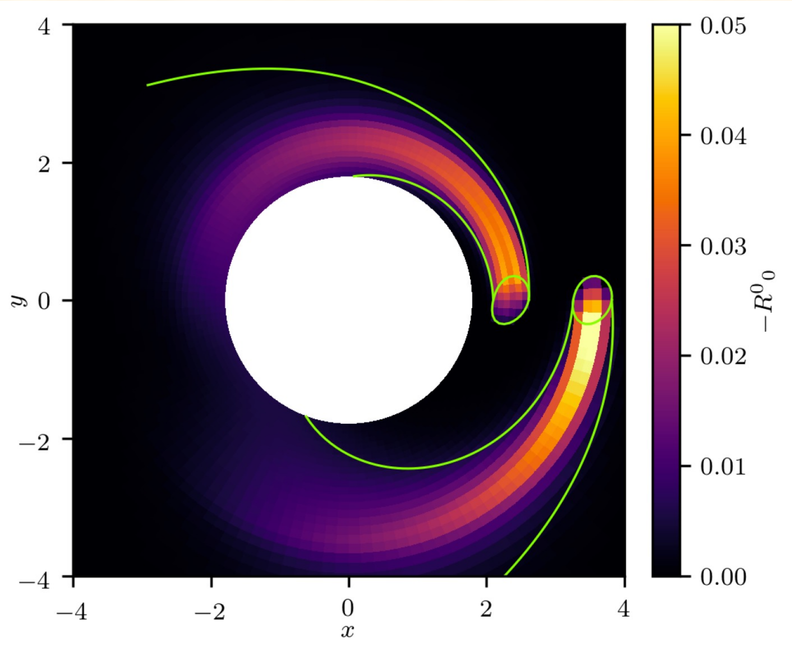
Spacetime plots



See also Blaes et al. (2006); Hirose et al. (2009)

Direct Methods: Discrete-ray transport

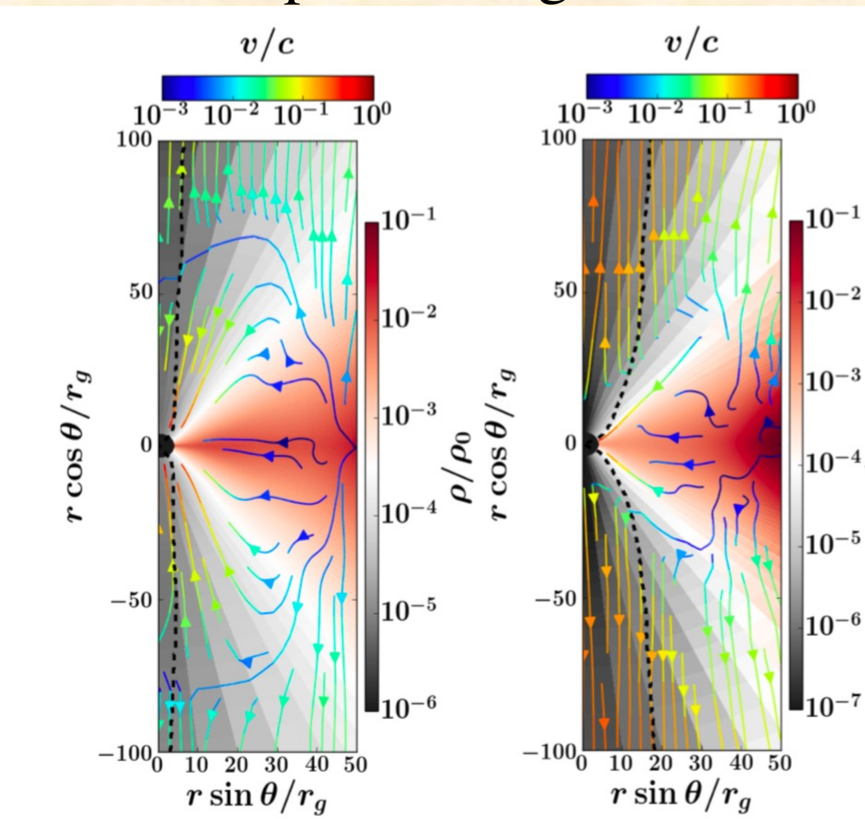
- Discretize I_ν over solid angle in each gid point, either using mesh points (Jiang et al. 2014b) or spectral basis (Radice et al.).
- Solve the time-dependent transfer equation directly for every angle over entire spatial mesh.
- Requires special techniques to propagate photons along geodesics between neighboring cells (C. White, in preparation)
- <u>Memory</u> and compute intensive



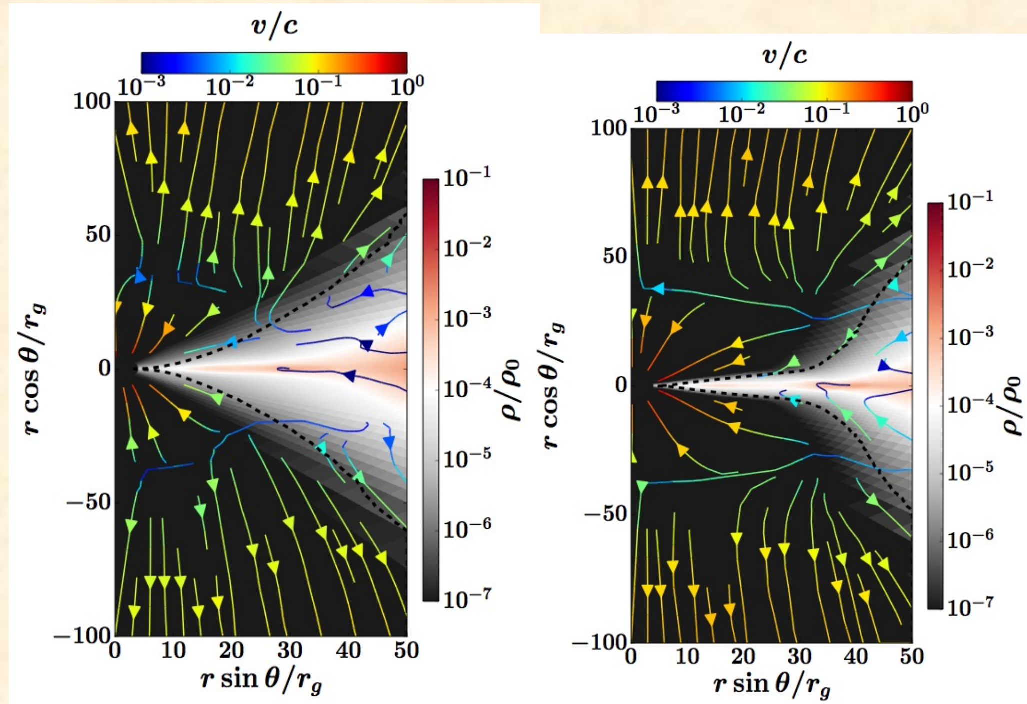
Propagating beam test in Kerr spacetime (courtesy C. White)

Method has enabled global simulations of radiation-dominated disks (Jiang et al. 2019a; 2019b)

Super-Eddington



sub-Eddington



Photosphere = black dashed line.

$\dot{M} = 150\dot{M}_{\rm Edd}$ $\quad$ $\dot{M} = 33\dot{M}_{\rm Edd}$ $\quad$ $\dot{M} = 0.2\dot{M}_{\rm Edd}$ $\quad$ $\dot{M} = 0.07\dot{M}_{\rm Edd}$

$M_{\rm BH} = 5 \times 10^8 M_\odot$

Sub-Eddington disks are thin with outflows, but outflow rate is very low.

Direct Methods: Monte Carlo

- The invariant intensity (I_ν / ν^3) is directly proportional to the photon distribution function.
- Discretize with Monte Carlo particles (photon packets) which propagate along geodesics and are emitted/absorbed/scattered.
- Take moments of the distribution function to compute radiation forces and net heating/cooling rate in the fluid.

Advantage: Simple, easy to incorporate complex microphysics
Disadvantage: Keeping noise below few percent is challenging.

Monte Carlo may be the best method for computing spectra in post-processing, and is becoming increasingly adopted for dynamics, e.g. Sedona (Kasen et al. 2006), bhlight (Ryan et al. 2015), SpEC (Foucart et al 2021).

Coupling radiation and fluid

Overall accuracy and stability of method determined by how material-radiation coupling terms handled. Generally implicit methods required.

- **Operator splitting:** transfer and fluid equations evolved separately. Simple, but at best first-order.
- **Picard iteration:** (Sekora & Stone 2010) Higher-order, but requires characteristic decomposition of quasi-linear form of evolution equations.
- **Implicit-Explicit (IMEX) methods:** (Pareschi & Russo 2005) modification to standard RK time-integration methods to include additional implicit stages. Only local implicit solves required, but still can be expensive.

Best strategy to achieve higher-order stable schemes still a matter of research.

3. Tests

Tests problems are crucial, but analytic solutions are generally hard to construct.

Tests of transport solver:

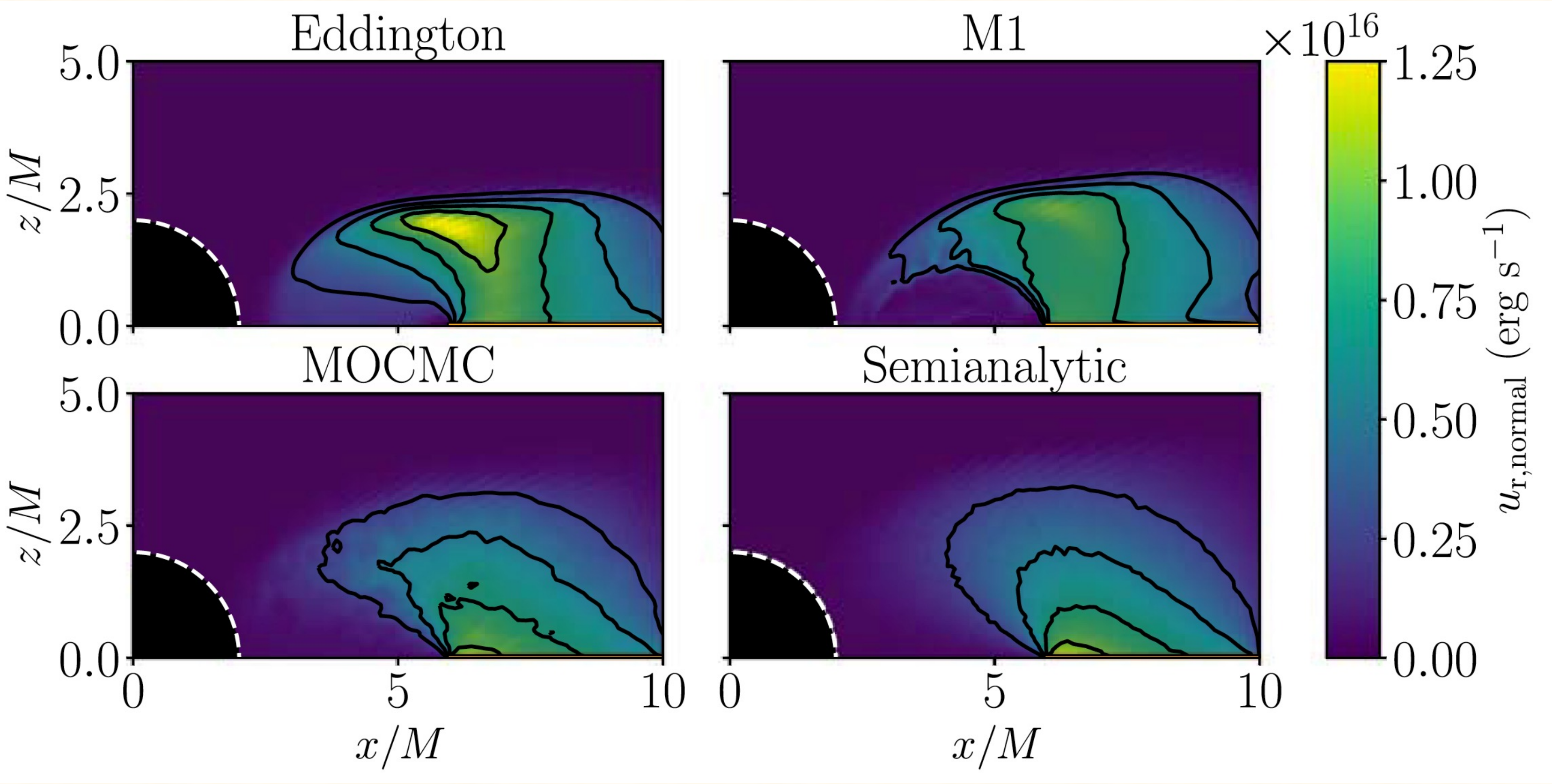
- Propagating beams
- Point sources
- Distributed sources

Tests of full coupled dynamics

- Linear waves (exact solution)
- Nonlinear shocks (semi-analytic solution)
- Instabilities (MRI, photon bubble)

Radiation field above hot, thin disk in Kerr.

Photons emerge from disk (flat plate) for 6 < *x*/*M* < 10
(Ryan & Dolence 2020)



Radiation energy density in normal observer frame

Full dynamics: Linear Waves

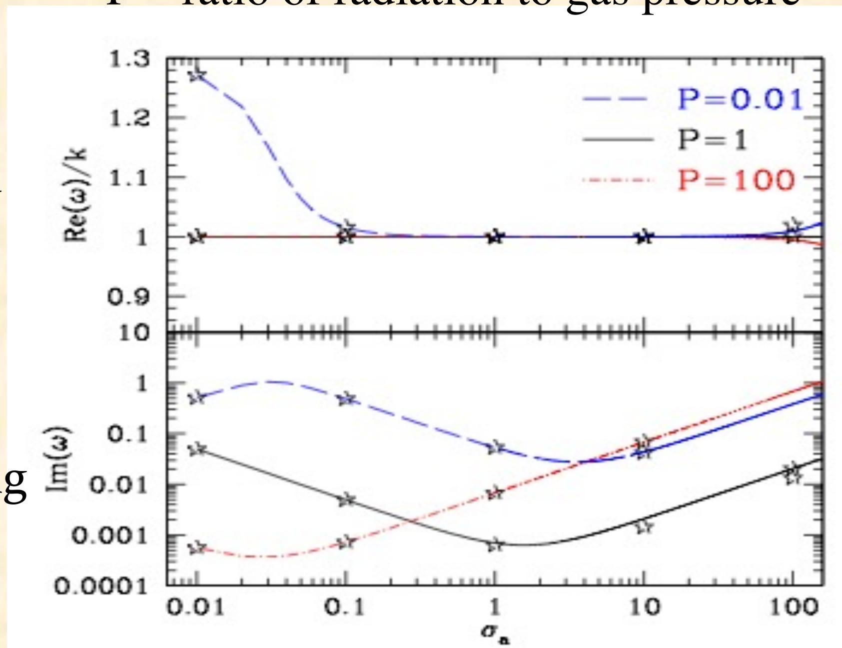
Quantitative measure of error and convergence rate.

P = ratio of radiation to gas pressure

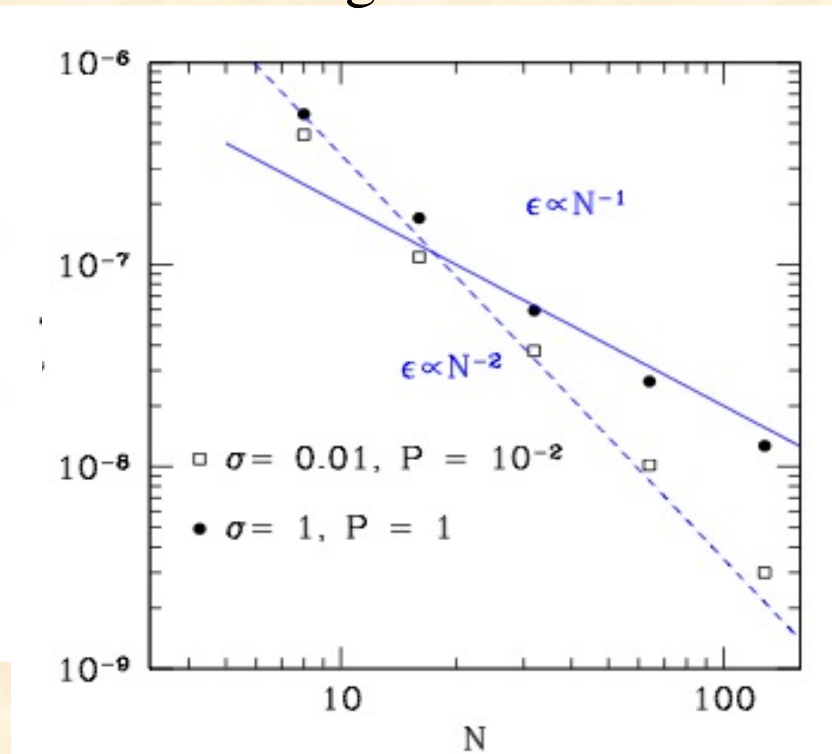
Convergence rate in 3D

Phase Velocity

Damping Rate



Optical depth per wavelength

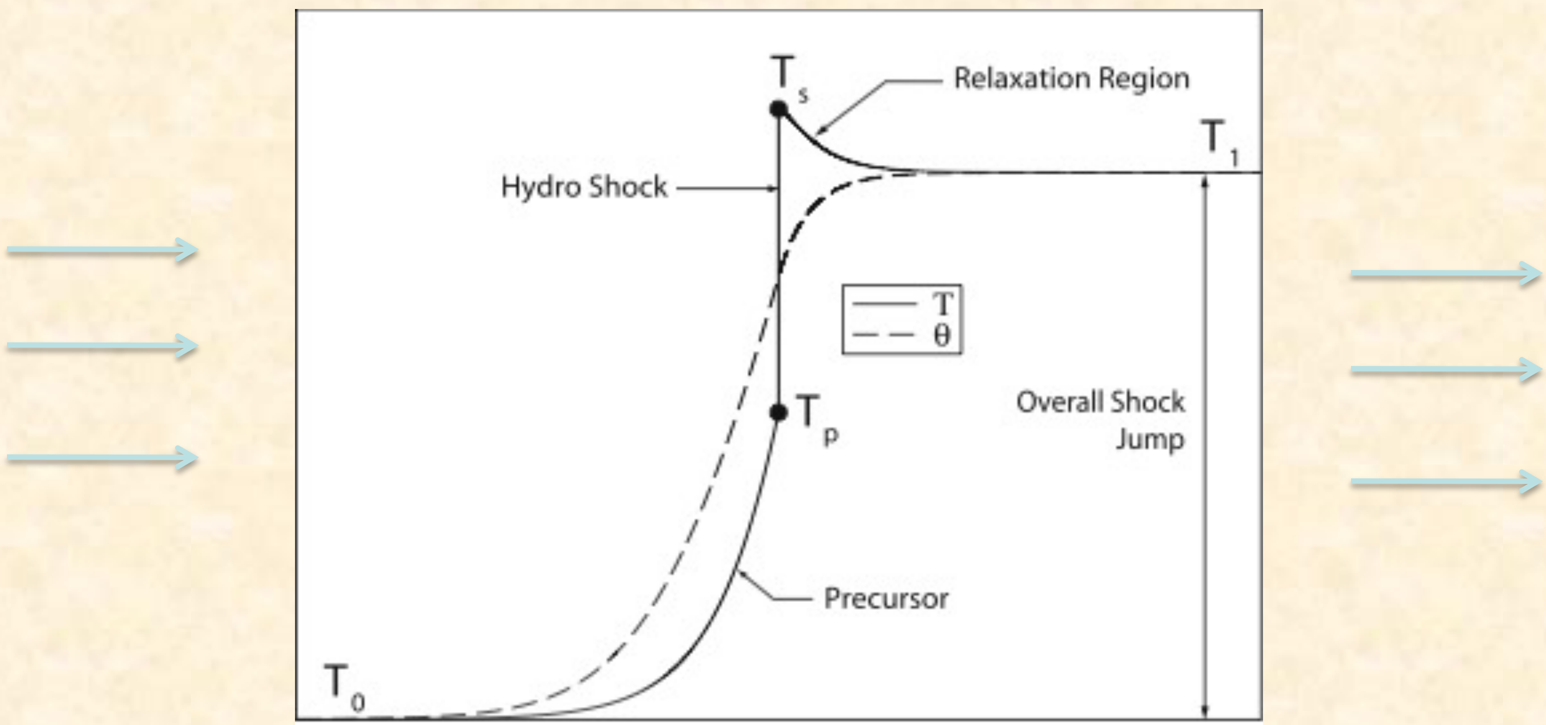


Stars are measured phase velocity and damping rate from 1D code.

Jiang et al. 2012

Radiation Shock Tests

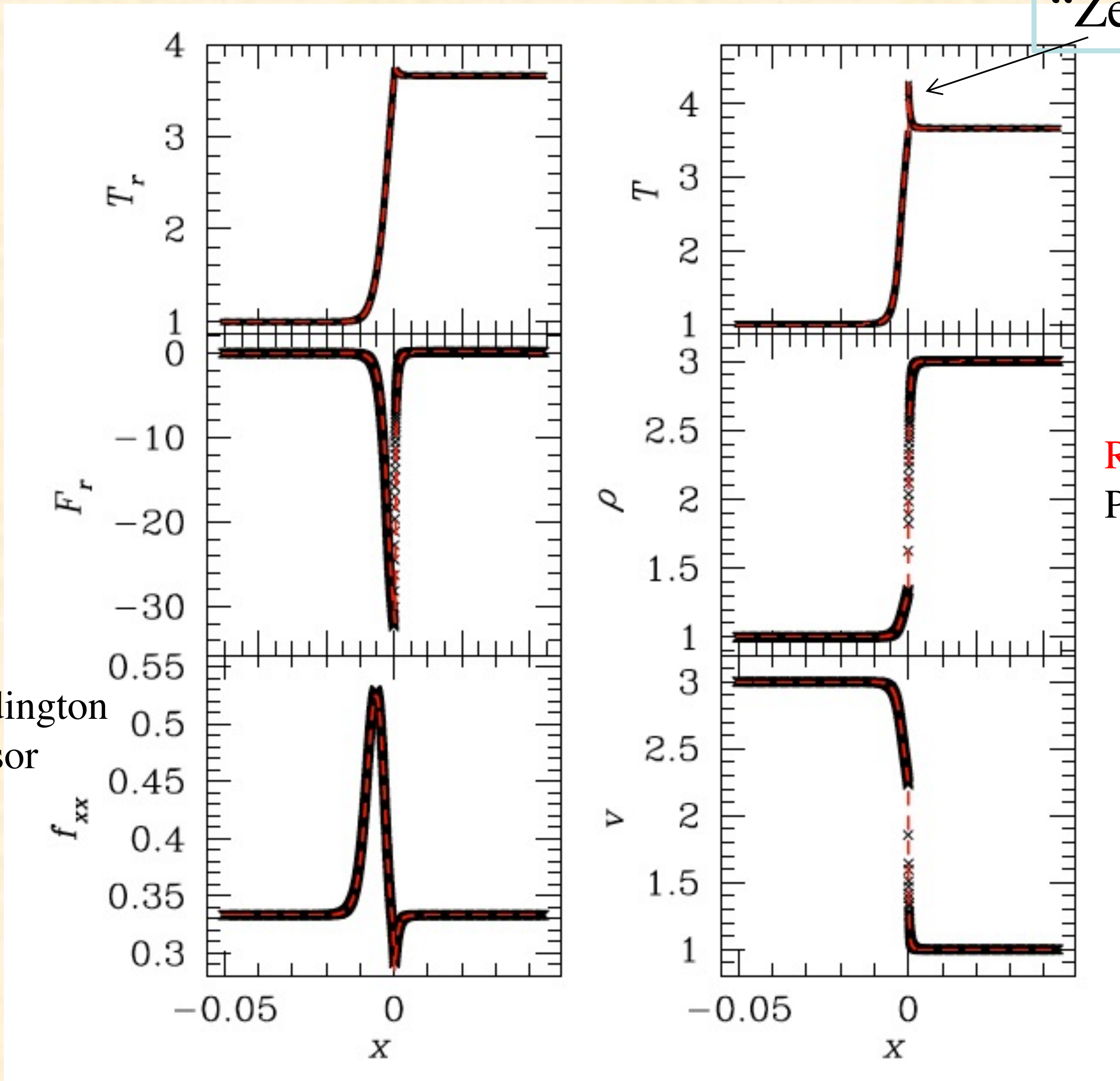
- 1D steady shock with pure absorption opacity
- Semi-analytic solution possible, Lowrie & Edwards (2008)



Shock structure changes with different Mach numbers.

Structure of Mach 3 shock

“Zeldovich spike”



Eddington tensor

Red line=reference sol’n
Points=numerical sol’n

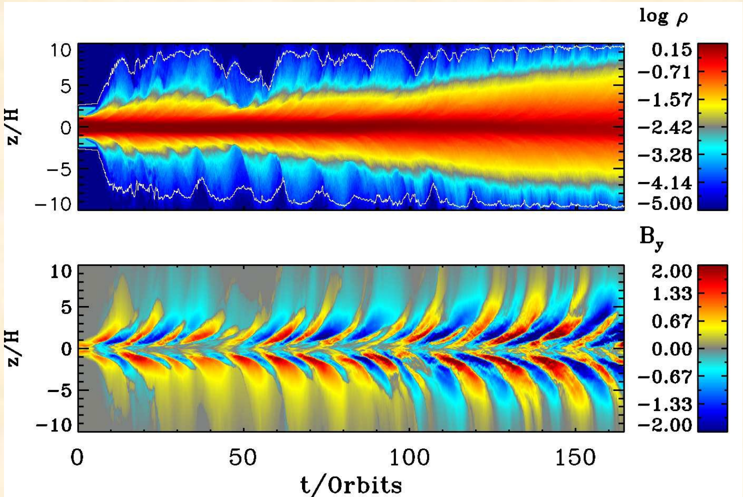
Jiang et al. 2014b

4. Challenging Issues

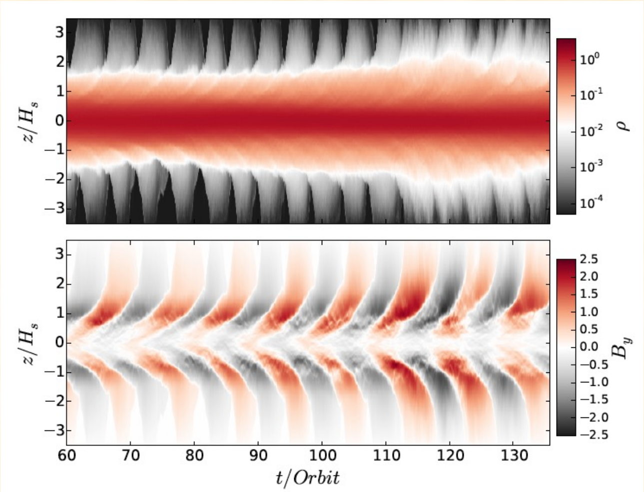
Issue #1: Dynamics depends sensitively on opacities.

Example: including the iron opacity bump at ~200,000K changes thermal stability in local shearing-box simulations of radiation pressure dominated disks ($P_{rad}/P_{gas} = 4.13$), Jiang et al. (2016)

Space-time diagrams



Electron scattering opacity only.



Including iron opacity.

Similarly, proper treatment of Compton cooling important for computing temperature in corona of MRI-unstable disks (Kinch et al. 2020)

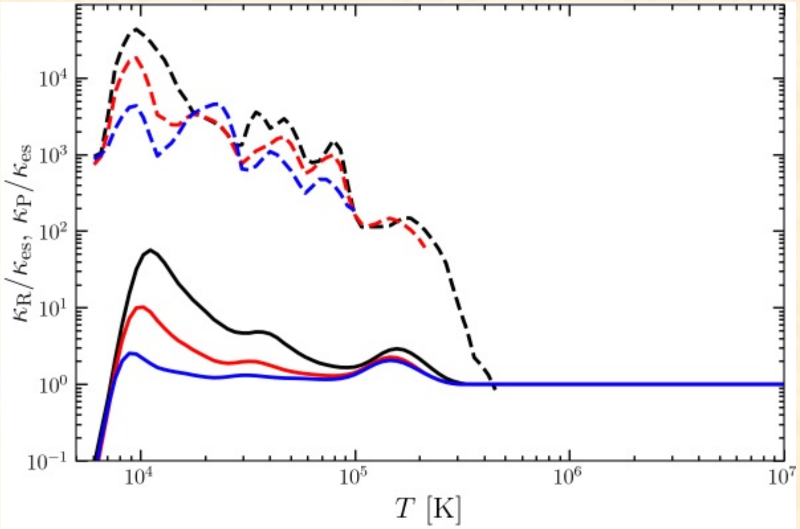
Issue #2:
Frequency integrated opacities are not always sufficient.

Typically, opacity tables provide either the Rosseland mean opacity (for absorption/scattering terms):

$$\frac{1}{\kappa_{\rm Ross}(\rho,T)} = \frac{\int d\nu \frac{\partial B_\nu(T)}{\partial T} \frac{1}{\kappa(\rho,T)}}{\int d\nu \frac{\partial B_\nu(T)}{\partial T}}$$

or Planck mean opacity (for emission terms): $\kappa_{\rm P}(\rho,T) = \frac{\int d\nu B_\nu(T) \kappa^{\rm abs}(\rho,T)}{\int d\nu B_\nu(T)}$

There can be an orders of magnitude difference between the two.



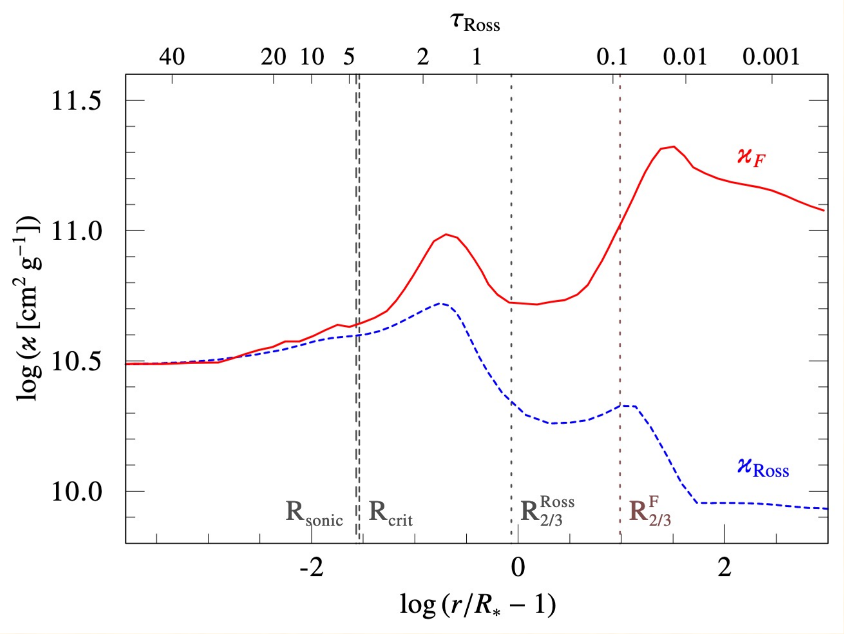
Solid lines – Rosseland mean
Dashed lines – Planck mean
T = 10^8, 10^9, 10^{10} K

(Figure by S. Davis)

Moreover, what is really needed is the flux-mean opacity:

$$\chi_F = \frac{\int d\nu F_\nu(T)\kappa(\rho,T)}{\int d\nu F_\nu(T)}$$

Sander, Vink, & Hamann (2019) demonstrate significant difference between flux- and Rosseland mean opacities in WR winds



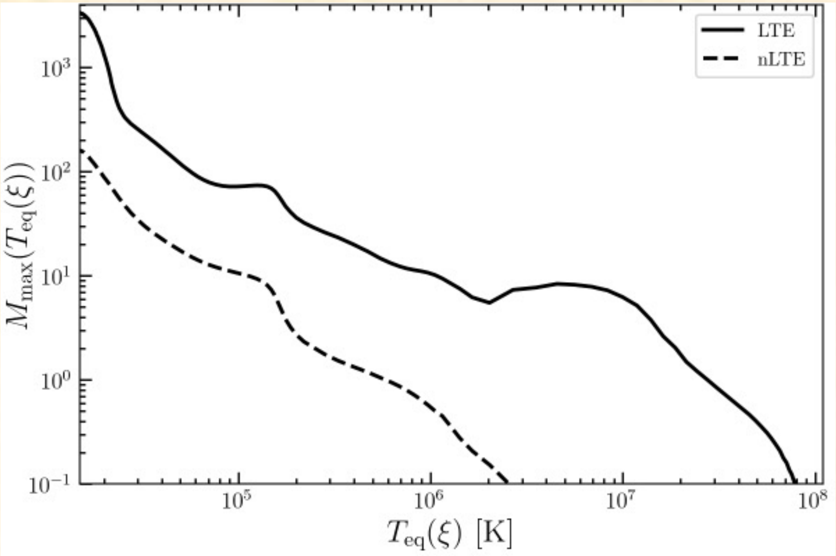
Calculations should include both frequency-dependent transport and opacities.

Issue #3:

Photoionization and non-LTE effects are important.

Radiation force in a partially ionized plasma can vary by orders of magnitude depending on whether or not non-LTE effects are taken into account:

Force-multiplier as a function of temperature for fixed photo-ionization parameter ξ (Dannon et al. 2019)



Corollary: usual CAK force multiplier cannot be used to model line driving in 3D turbulent flow. More general approximations are required.

Issue #4:
Leveraging exascale.

As of June 2021, 7 of the 10 fastest computers in the world rely on GPUs or other accelerators. Programming heterogeneous systems is complex.

Fortunately, open source tools for ***performance portability*** are available, e.g. Kokkos https://github.com/kokkos/

Athena++ has been re-written in Kokkos with very encouraging results. Excellent performance and scaling on both CPUs and GPUs.

Athena++ AMR infrastructure in Kokkos available through the `Parthenon` project: https://github.com/lanl/parthenon

5. Summary

- Realistic models of most accretion flows require including radiation transport.
- Numerical methods for RT is an active area of development. Direct methods are now feasible.
- Better treatment of radiation-material interaction processes (including realistic frequency-dependent opacities and non-LTE effects) is a challenge for the future, and is probably crucial for some problems.